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ANALYTICAL DESIGN METHODS FOR AIRCRAFT STRUCTURAL JOINTS

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DALLAS, TEXAS

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FOREWORD

This report was prepared by the Vought Aeronautics Division of the LTV Aerospace Corporation, Dallas, Texas, under USAF Contract F33615-67-C-1339. The work was initiated under Project No. 1467 "Structural Analysis Methods", and Task No. 146704 "Structural Fatigue Analysis". The work was administered under the direction of the Air Force Flight Dynamics Laboratory, Directorate of Laboratories, Wright-Patterson Air Force Base, Ohio. Mr. Howard A. Wood was technical monitor.

This report covers work conducted from 31 January 1967 through 31 January 1968. Mr. W. F. McCombs was Principal Investigator. Technical assistance was provided by Mr. J. C. McQueen who developed the computer routines. Mr. J. L. Perry was test engineer in charge of the fabrication and testing of all specimens and of the photostress analyses. Consulting services were provided by Dr. R. L. Tucker, Professor of Civil Engineering, University of Texas at Arlington, Texas. This report was submitted by the authors on 31 January 1968.

This technical report has been reviewed and is approved.

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Chief, Theoretical Mechanics Branch
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ABSTRACT

An engineering procedure for determining the distribution of loads in the mechanically fastened joints of splice and doubler installations has been developed. Methods for both hand analyses and computer analyses are presented. Routines for solution by digital computer are provided.

The methods are generally limited to the cases of a single lap arrangement and a single sandwich arrangement, but the case of multiple (stacked) members is discussed. The members may have any form of taper or steps and the effects of fastener-hole clearance, or "slop", and plasticity can be accounted for. The particular primary data that must be supplied but which are not generally available in the literature are the spring constants of the fastener-sheet combinations.

A test program has been carried out to substantiate the methods and the results are included.

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NOMENCLATURE, SYMBOLS AND DEFINITIONS

A	area of a cross-section
B	a ratio of two thicknesses
C	constant of integration
D	designation for an axial member, either a doubler or the upper member in a splice; also used to designate a hole diameter
E	modulus of elasticity
e	natural logarithm base; also designates an eccentricity
f_t	a tensile stress
f_c	a compressive stress
f_s	a shear stress
F	an allowable stress
G	modulus of elasticity in shear
h	dimension involving thicknesses of axial members and the bond
k	spring constant of a member or of a fastened joint
k_o'	the "secondary" spring constant of a fastened joint obtained in unloading or reloading the joint.
L	the length of a member, or of an element of a member
m	a subscript referring to the number of a set of calculations within a larger set.
n	a subscript referring to the number of a member or of a calculated value
p	fastener spacing (or "pitch")
P	internal load
q	internal shear flow
q_a	applied shear flow
Q	applied axial load
r	a ratio of loads

R	an external reaction
S	designation for a base structure member, or the lower member in a splice
t	a thickness
T	a tension or compression load in a direction normal to the applied axial loads.
U	strain energy
w	normal running load (lbs/in.)
W	width of an axially loaded member
x	coordinate in the direction of the axial load
z	coordinate normal to x (or "vertical")
δ	the total strain in a member (or in an element of a member) or in a fastened joint; referred to as the "deflection" in a fastened joint
Δ	an increment
ν	Poisson's ratio
Δc	the initial clearance or "slop" in a fastened joint.

SECTION I

INTRODUCTION

I.1 GENERAL

There are numerous occasions both in the design stages and in the service life of aerospace vehicles when it may be desirable to use either splices or doublers (reinforcing members) having many rows of fasteners in the direction of the applied loading. The proper, or the optimum, arrangement of such members requires a definition of the loads transferred by the various fasteners. To be practical this definition of loads must also reasonably account for possible fastener-hole clearance (or "slop") and for loadings that carry the joints into the plastic range. Once defined, the fastener loads can be used to assess the structure for adequacy under any general criteria. That is, where a stipulated fatigue life is a requirement the local fastener bearing stresses on the members must be small enough so as not to result in an unacceptable fatigue life limitation. And, where yielding and/or ultimate strength are the criteria, the fastener loads must be small enough that these are satisfied. Finally, any such methods of analysis should be useable for a hand analysis of specific structures. That is, even though a computer program is available and even though some "idealization" of the structure may be necessary, the advantages of hand analyses can be numerous in many instances.

I.2 LITERATURE SURVEY

A considerable number of published papers, reports and textbooks containing discussions related to the subject of this report have been reviewed. These are listed in the Bibliography. Those which appear to be most pertinent for this effort are listed as References and are referred to in the applicable section of this report. In general it was found that most discussions were for spliced members having a bonded joint, a few were for spliced members with bolted or riveted joints, but none were found for the case of the installation of a doubler. Where outlined, most methods were limited to the elastic range, the members and attachments were uniform (no taper or steps), the effect of fastener-hole clearance was not included and, importantly, no significant data defining the stiffnesses (or the "spring constants") of the fastener-sheet joints appears to be in the literature. Summarizing, the present literature does not appear to provide the engineer with suitable general methods and data necessary for proceeding with the analyses of doubler and splice installations having mechanically fastened joints. A brief description of these references follows.

Reference (1) makes use of a large rubber analog (model) for measuring and actually observing, by marked grid-lines, the displacements taking place in a cemented and in a riveted joint. The report is interesting in that it gives a better insight as to the physical manner in which such joints actually deform. A theoretical analysis

for a cemented joint is presented and the results obtained by using it were verified from tests of the model. The tension forces across the joint, as well as the shear distribution were discussed. No qualitative data or methods were presented, however, that could be used directly for predicting the load distribution in a mechanically fastened joint. The analysis presented uses the elementary theory and is for the lap splice only. The effects of fastener "slop" and plasticity are not included.

Reference (3) is generally referred to as the "exact" analysis of a bonded lap splice. Equations are developed for the shearing and "tearing" (tension) stresses in the bond. The equations are quite lengthy and involve hyperbolic functions. The extreme cases of a relatively flexible bond and of a "rigid" bond are evaluated. The members are uniform (no taper). The results are of interest primarily for the case of short bonded lap splices, rather than for mechanically fastened joints.

Reference (4) discusses the analogy between the distribution of current in a ladder-type resistance network and the distribution of loads in a bolted joint (and also in stiffened panels). A simple "computer" consisting of variable resistors and a constant current source was described. It s use was shown to give a very rapid determination of bolt loads with an accuracy quite acceptable for engineering design. Such a simple computer would be especially useful where long joints are involved and also where unsymmetrical structural arrangements are present. It would also serve to define load distributions in stiffened panels where shear-lag effects are present.

Reference (5) presents (as part of a larger effort) a computer program for the determination of fastener loads in a splice having multiple axial members. The program is based upon the elementary theory and arrives at the fastener loads by solving simultaneous equations. Hence, it is not useful for hand analyses. This reference is discussed further in Section IV.

Reference (6) is the first major effort published by the NACA on the subject. Only the symmetrical case is discussed, however. An equation for determining the spring constants of bolts in double shear and in the elastic range is presented. The method consists of using an equation developed for the load relationship between adjacent fasteners to obtain the loads in all of the fasteners in the elastic range. Hence, as presented, the method is restricted to bolted symmetrical butt joints in the elastic range. No consideration is given to unsymmetrical arrangements, bolt hole clearance, or stresses above the elastic range. Tests were carried out which verified the results of the method.

Reference (7) is an extension of the earlier work in Reference (6). It consists essentially of developing a "recurrence formula" which can be used, with the appropriate boundary conditions, to rapidly

write simultaneous equations for the bolt loads. Then, to avoid the solution of simultaneous equations, a method of solution by a finite-difference equation is presented for uniform bolt size and spacing. This enables the direct solution of each bolt load to be obtained. The analogy between the bolted joint problem and the shear lag problem was mentioned and the shear-lag equation for single-stringer structures (NACA Report 608) was used to obtain the individual bolt loads. The main advantage of this method over the earlier effort is a saving of computational labor when a long joint with many fasteners is involved. It, too, is restricted to bolted symmetrical butt joints in the elastic range and also to uniform bolt size & spacing for the special techniques. Tests were carried out which verified the results obtained by the calculations.

I.3 SCOPE AND APPLICATIONS

The purpose of this effort is to provide the engineer with useable methods for determining the load distributions in any practical structural splice or doubler arrangement. The methods are generally restricted to a single lap or to a single sandwich (3 axial members) but it is believed that this covers the majority of practical cases likely to be encountered. The effects of both fastener hole clearance ("slop") and plasticity can be accounted for. The load distributions can be calculated either by hand analyses or by using either a digital or an analog computer There are two types of hand analyses. One type (Method 1) uses theoretical formulas that are strictly applicable only for the case of uniform members in the elastic range and does not account for fastener-hole clearance. The other type of hand analysis (Method 2) is a numerical procedure and hence applies to any case since the effects of taper, fastener-hole clearance and plasticity are accounted for. The results of a test program carried out to assist in defining parameters and to substantiate the method are presented.

The use of splices in aerospace vehicle structures is well known. It is accepted as "good design practice" to use a minimum number of rows of attachments in designing splices, but there are occasions when such practice cannot be observed and many rows are required. It is in these cases, particularly, that an accurate determination of the individual fastener loads is necessary.

The use of doublers in aerospace vehicle structures would possibly be made for any of several general purposes which are

a. Reinforcement for strength purposes in order to

(1) strengthen an existing structure

(2) salvage a damaged area

(3) strengthen an axially loaded member having a "cut-out"

In any case the possibility of a limitation in fatigue life due to such a doubler installation should be considered as a possible unacceptable limitation. If this is no problem, then either a yielding or strength capability is the main criteria.

b. Reinforcement for fatigue purposes in order to:

(1) increase the life of an existing design

(2) properly salvage a damaged structure from a service life consideration.

(3) salvage a "fatigue damaged" structure (i.e. where, fatigue damage has been accumulated too rapidly in a particular vehicle or group of vehicles)

c. Reinforcement for stiffness purposes which should include a consideration of a possible fatigue life limitation.

d. Although not necessarily intended as such, any member attached to an axially loaded structure will act as a doubler, picking up load. In such cases an investigation of possible harmful effects on fatigue life is sometimes desirable or necessary.

e. An additional application of the method is in investigating the possible consequences of ending a member, such as a stringer, that is attached to a skin or sheet. Occasionally such practice may be desirable from the manufacturing or salvage standpoint, and any possible harmful consequence will require analysis.

Summarizing, it is believed that this report provides the engineer with practical methods for proceeding with the analyses of mechanically fastened joints. The fastener data necessary for such analyses are discussed and some typical data are presented.

SECTION II

METHOD 1 - ANALYSIS BY THEORETICAL FORMULAS

II.1 INTRODUCTION

The purpose of this section is to present the development of formulas that can be used to predict load distributions in various splice and doubler configurations. The formulas will give approximate predictions since they are obtained from elementary principles and simplifying assumptions. However, they are useful for making engineering estimates for the cases to which they apply. It appears that any attempt to use other than an elementary approach results in expressions that are not of a useable form for design purposes. Also, the available data for the installed fasteners does not warrant such a refinement in analysis at present. (Such is not the case for bonded joints, however, where some provision in analysis must be made to account for the tension stresses in the bond at the ends of the joint. This particular stress is not accounted for by the elementary theory).

Although the numerical methods of Section III are the ones that will actually be used by the engineer in nearly all practical cases, it appears to be quite helpful for him to have an understanding of the elementary theory including its limitations and applicability. This is presented in Section II.

II.2 ELEMENTARY THEORY

The following analysis is based on several specific assumptions. Referring to Figure II.1 which represents a doubler installation:

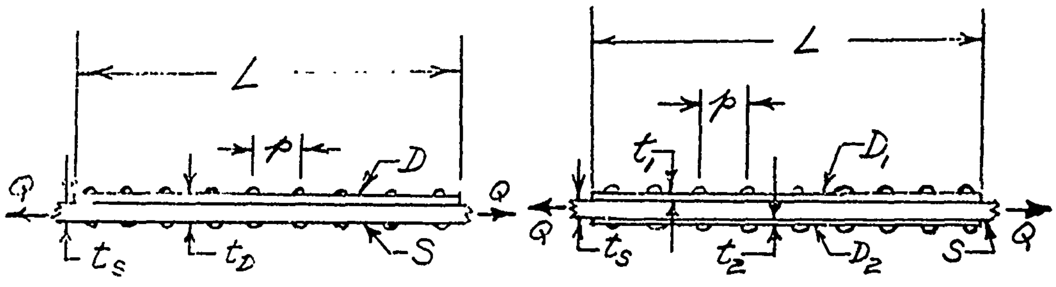


Figure II.1. Types of Doubler Installations Analyzed

a. There are only 2 joint configurations to which the analysis applies

   (1) a single lap as in Fig. II.1a

   (2) a single sandwich as in Fig. II.1b

   (The same would apply to splice configurations)

b. All stresses are in the elastic range.

c. The axial members, S and D are each of uniform size, no taper or steps.

d. The axial members are subject only to uniform axial stress (no bending stresses). Bending effects are discussed in Section VI *.

e. The fasteners are of a uniform size and are at a uniform spacing, p.

f. The fasteners have a spring constant in shear, k_f, obtain- from experimental load-deflection data for particular sheet thickness, t_s and t_D. These are discussed in Section VII. These discrete spring constants can be replaced by an "equivalent bond" having a shearing spring constant per inch of length given by

$$k = \frac{k_F}{p}$$

where p is the fastener spacing.

A sandwich configuration as in Fig. II.1b can then be analyzed in the same manner as the configuration in Figure II.1a by combining the separate members D_1 and D_2 into one member D (having their total cross-sectional area) and using the spring constant, k_F, that corresponds to the actual double lap fastener sheet combination in determining the value of k for the single bond.

Thus, an arrangement consisting of a base structure, S, subjected to the applied axial load Q and having (either one or two) doublers installed, as shown in Fig. II.2a, (and II.1b) can be analyzed using the equivalent structure shown in Figure II.2b. Due to symmetry the structure can be further simplified as shown in Figure II.2c.

* In Ref. 6 (Bolted Sandwich Splices) it is shown that the bending has a negligible effect upon the distribution of fastener loads.

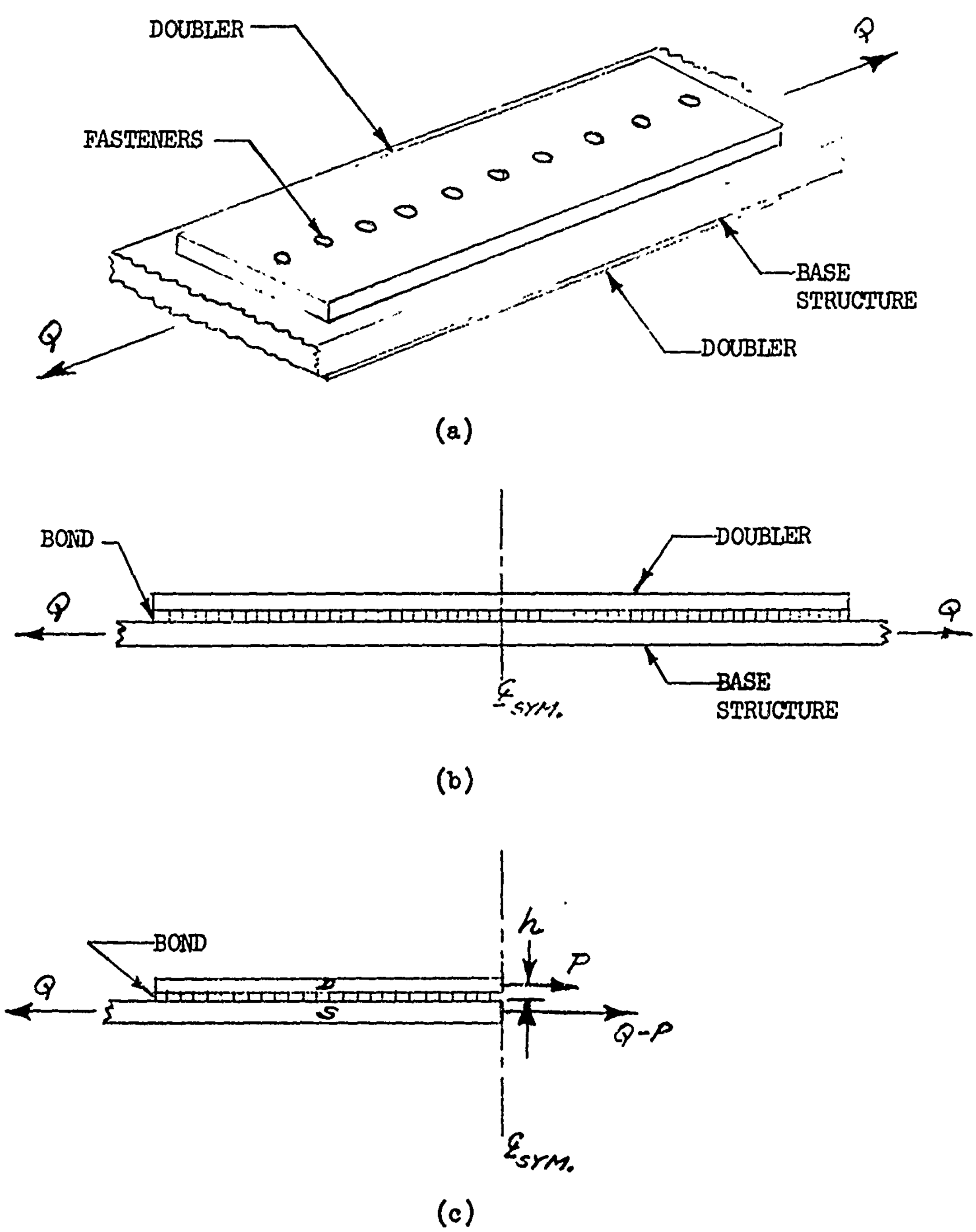


Figure II.2 Conversion Of Doubler Installation Into Its Equivalent Structure

Since an equivalent bond is being used, the results will of course also apply to members which are actually bonded together.

As the member S stretches under the load Q the member D will be caus' ' to stretch also, because of the common bond (or the attachments). A d, P, will thus be developed in the member D, varying from zero at the ends to a maximum at the centerline. At any station the net load in the base structure will then be Q less the load in D.

Referring to Figure II.3, the load in the doubler at any station, x, can be determined as follows, using the previously listed assumptions.

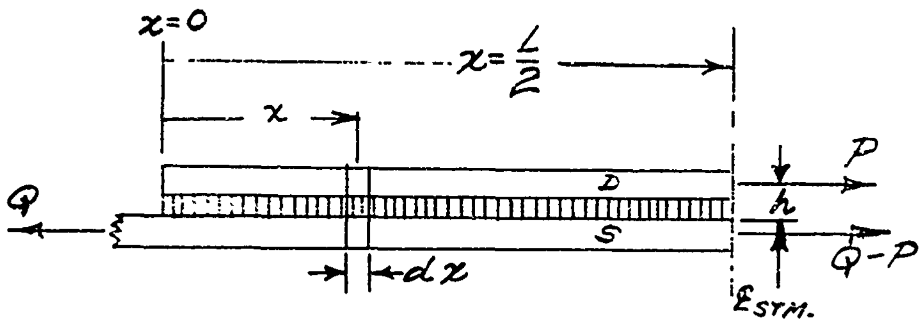


Figure II.3 One Half Of A Doubler Installation

From the minimal energy principle the variation of the load P must be such as to result in a minimum of energy being stored in the structure as a whole. There are, per the assumptions, three sources of stored energy, U . These consist of axial strain energy in the members D and S and shear strain energy in the bond, or

$$U = U_D + U_S + U_B$$

In differential form, for an element of length dx,

$$dU = dU_D + dU_S + dU_B$$

where

$$dU_D = \frac{P^2 dx}{2 A_D E_D}$$

$$dU_S = \frac{(Q-P)^2 dx}{2 A_S E_S}$$

$$dU_B = \frac{(dP)^2}{2K} = \frac{\left(\frac{dP}{dx} \cdot dx\right)^2}{2k\, dx} = \frac{\left(\frac{dP}{dx}\right)^2 dx}{2k}$$

Hence,

$$dU = \left[\frac{P^2}{2A_D E_D} + \frac{(Q-P)^2}{2A_S E_S} + \frac{1}{2k}\left(\frac{dP}{dx}\right)^2\right]dx \text{ -------- } (1)$$

And,

$$U = \int_0^{L/2} \left[\frac{P^2}{2A_D E_D} + \frac{(Q-P)^2}{2A_S E_S} + \frac{1}{2k}\left(\frac{dP}{dx}\right)^2\right]dx \text{ ------ } (2)$$

Referring to the bracketed terms in Eq (1) and (2) as F, Eq.(2) becomes

$$U = \int_0^{L/2} F\,dx \text{ ---------------- } (3)$$

It is shown in the literature, Reference (2), that when F is a function of the variables P and dP/dx , the particular manner in which P must vary with x in order to minimize the integral as in Eq.(3) is defined by the equation

$$\frac{\partial F}{\partial P} - \frac{d}{dx}\left(\frac{\partial F}{\partial \frac{dP}{dx}}\right) = 0 \text{ -------------- } (4)$$

Eq. (4) is usually referred to as "Euler's Equation"

Therefore in order to apply Equation (4) to Equation (2), the derivatives are first obtained, from Equation (2), as

$$\frac{\partial F}{\partial P} = P\left(\frac{1}{A_S E_S} + \frac{1}{A_D E_D}\right) - \frac{Q}{A_S E_S}$$

$$\frac{\partial F}{\partial \frac{dP}{dx}} = \frac{1}{k}\frac{dP}{dx}$$

$$\frac{d}{dx}\left(\frac{\partial F}{\partial \frac{dP}{dx}}\right) = \frac{1}{k}\frac{d^2P}{dx^2}$$

Then, substituting these terms into Equation (4)

$$P\left(\frac{1}{A_S E_S}+\frac{1}{A_D E_D}\right)-\frac{Q}{A_S E_S}-\frac{1}{k}\frac{d^2 P}{dx^2}=0 \quad \text{------}(5)$$

Rearranging terms

$$\frac{d^2P}{dx^2}-k\left(\frac{1}{A_S E_S}+\frac{1}{A_D E_D}\right)P=-\frac{kQ}{A_S E_S}$$

or

$$\frac{d^2P}{dx^2}-MP=-N \quad \text{-------------}(6)$$

$$\text{where} \quad M=k\left(\frac{1}{A_S E_S}+\frac{1}{A_D E_D}\right) \quad \text{and} \quad N=\frac{kQ}{A_S E_S}$$

The solution of (6) gives the doubler load P as a function of x

$$P=C_1 e^{\sqrt{M}x}+C_2 e^{-\sqrt{M}x}+\frac{N}{M} \quad \text{------}(7)$$

The constants C_1 and C_2 are determined from the end conditions, which are, for this case,

$$\text{At } x=0,\ P=0 \text{ and at } x=\frac{L}{2},\ \frac{dP}{dx}=0$$

This results in

$$C_1=-\frac{N/M}{1+e^{\sqrt{M}L}} \quad \text{and} \quad C_2=C_1 e^{\sqrt{M}L}$$

Hence

$$P=C_1\left(e^{\sqrt{M}x}+e^{\sqrt{M}L}\cdot e^{\sqrt{M}x}\right)+\frac{N}{M} \quad \text{--------}(8)$$

Equation (8) thus defines the doubler load at any station x.

The shear flow, q, at any station, x, can then be obtained by differentiating (8), giving *

$$q=\frac{dP}{dx}=\sqrt{M}\,C_1\left(e^{\sqrt{M}x}-e^{\sqrt{M}L}\cdot e^{\sqrt{M}x}\right) \quad \text{------}(9)$$

and in a similar manner the tension on the bond (normal to the applied load) can be obtained at any station x <u>except</u> the end by differentiating Equation (9), and multiplying by the distance h, giving *

$$w=h\frac{dq}{dx}=hMC_1\left(e^{\sqrt{M}x}+e^{\sqrt{M}L}\cdot e^{-\sqrt{M}x}\right) \quad \text{-----}(10)$$

where h is the distance between the centroid of D and the inner surface of S as in Figure II.3.

The actual shear load, P_F, on a fastener at any station x can be obtained as (approximately)

$$P_F=q_x\cdot p$$

* See Figure II.5

where p = fastener spacing

q_x = shear flow from Eq. (9)

For the end fastener, however, the shear flow is usually changing so rapidly that it is more accurate to use Eq. (8) with x = p to obtain P_{F1}. That is, $P_{F1} = P_{x=p} - P_{x=0} = P_{x=p}$

Although Equations (8) and (9) are somewhat lengthy, the designer or analyst using them would only be interested in calculating the value of P at one station, at x = L/2, and in calculating the value of the end fastener load. Hence, not a great deal of computational labor is actually involved. And even this can be shortened by reducing these particular equations to the approximate expressions

$$P \approx \frac{N}{M}\left(1 - e^{-\sqrt{M}x}\right) \quad \text{-------------} \quad (11)$$

$$q \approx \frac{N}{\sqrt{M}} e^{-\sqrt{M}x} \quad \text{-------------} \quad (12)$$

which are sufficiently accurate for practical doubler installations. The larger the value of the parameter $e^{\sqrt{M}L}$, the more accurate are Equations (11) and (12). Then for the end fastener, P_{F_1}

$$P_{F_1} = \frac{N}{M}\left(1 - e^{-\sqrt{M}p}\right)$$

The results for other loadings on a doubler installation are summarized in Article II.6

II.3 ANALYSIS OF A SPLICE

Proceeding in a similar manner for a single lap splice (or for a single sandwich splice as mentioned previously) as illustrated in Figure II.4, the same differential equation, Equation (6), and general solution, Equation (7), are obtained

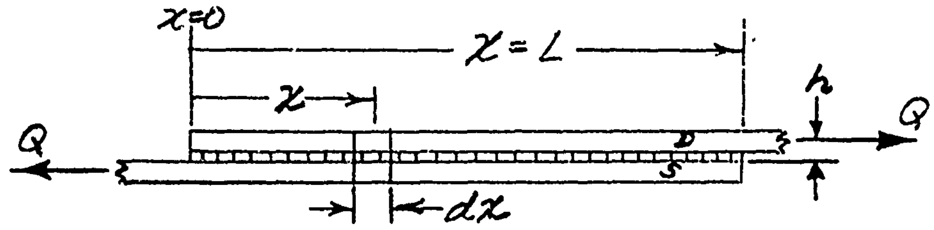


Figure II.4 A Splice

$$\frac{d^2P}{dx^2} - MP = -N$$

and

$$P = C_1 e^{\sqrt{M}x} + C_2 e^{-\sqrt{M}x} + \frac{N}{M}$$

where, as before, P is the axial load in member D. In this case, however, the end conditions are

$$At\ x = 0,\ P = 0 \quad and\ at\ x = L,\ P = Q$$

giving

$$C_1 = \frac{Q - \frac{N}{M}\left(1 - e^{-\sqrt{M}L}\right)}{e^{\sqrt{M}L} - e^{-\sqrt{M}L}} \quad and \quad C_2 = -\left(C_1 + \frac{N}{M}\right)$$

The resulting equations are then

$$P = C_1\left(e^{\sqrt{M}x} - e^{-\sqrt{M}x}\right) + \frac{N}{M}\left(1 - e^{-\sqrt{M}x}\right) \text{ ------- } (13)$$

$$q = \sqrt{M}\, C_1\left(e^{\sqrt{M}x} + e^{-\sqrt{M}x}\right) + \frac{N}{\sqrt{M}}\, e^{-\sqrt{M}x} \text{ ------- } (14)$$

and as discussed for Eq. (10),

$$w = hMC_1\left(e^{\sqrt{M}x} - e^{-\sqrt{M}x}\right) - Ne^{-\sqrt{M}x} \text{ ------ } (15)$$

These equations are somewhat lengthy, but, as discussed before, the designer would only be interested in obtaining the value of the end fastener load, (at the end of the larger member, S or D, where it is largest). This can be arranged by letting D be the larger member. Hence, only very little computational labor is involved. Equations (13)-(15) give the same results as their counterparts in Reference (1).

The results for other types of splices and splice loadings are presented in Article II.6. Although the various equations apply only to a configuration having uniform members, they can be used in making estimates for other cases. This is discussed in Article II.6. The main difficulty in practice is obtaining the values of k, as discussed in Section V. Example problems are presented at the end of this section.

II.4 EXTENDED ELEMENTARY THEORY

The previous elementary analysis considered only axial strain energy in the axial members and shear strain energy in the bond. The resulting static balance for, say, the splice of Figure II.4 is shown in Figure II.5.

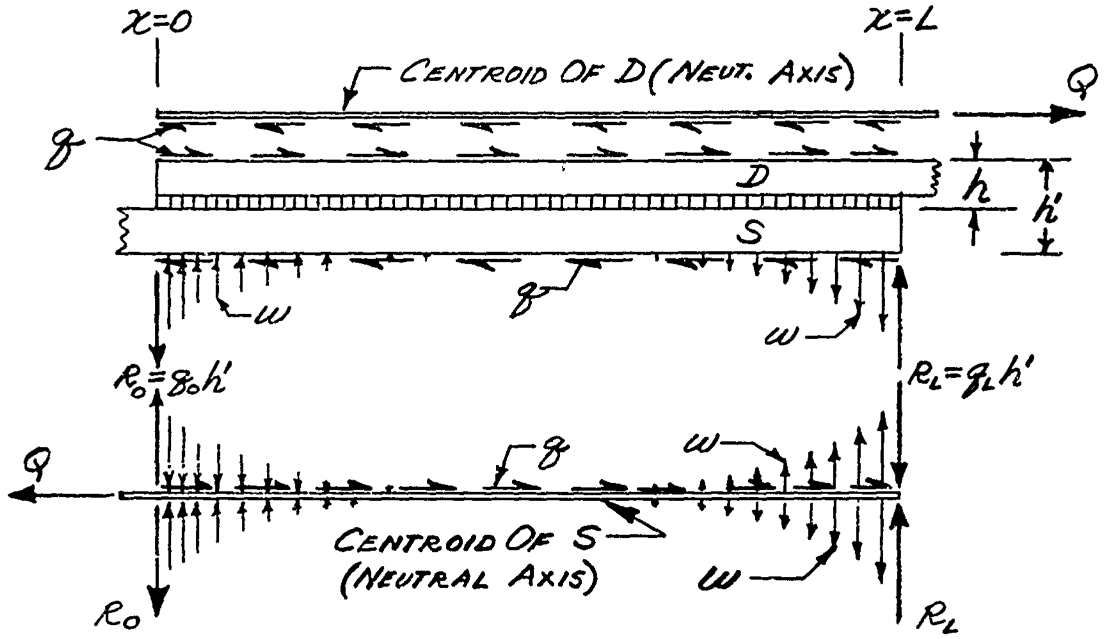


Figure II.5 Static Equilibrium of a Splice

The reactions, w and R, (which must be supplied per the assumptions) obviously will produce normal stresses in the bond which have been ignored. That is, any tension or compression energy in the bond has been assumed to be zero (or the bond is assumed infinitely rigid in this normal direction, as are the members S and D). It is of interest to see what the effect of including this energy would be on the final equations for P, q and w. This will also demonstrate how refining the elementary theory in even a simple manner results in expressions that are too involved for practical useage. Also, the results will apply only to an actual bonded (glued) joint rather than to a mechanically fastened one, as discussed later.

This particular effect can be accounted for by adding a fourth energy term to those of Equation (1), namely the normal force energy in the bond (which is, in practice, far greater than that in the normal direction for the stiffer members, S and D). Considering a small element dx as shown in Figure II.6,

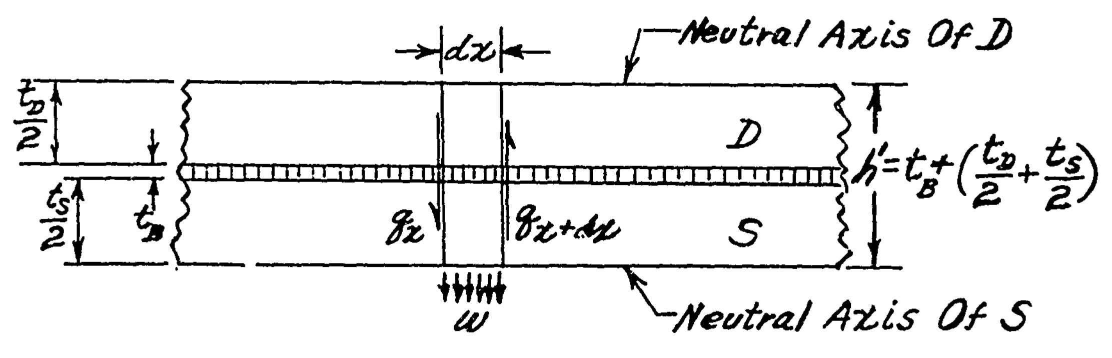


Figure II.6 Static Force Equilibrium of a Differential Element

For static equilibrium of forces in the normal direction,

$$q_x \cdot h = q_{x+dx} \cdot h' - w\,dx$$

$$h(q_{x+dx} - q_x) = h'\left(\frac{dq}{dx}\right)dx = w\,dx$$

The average normal load, T, in the bond can then be calculated as

$$T = B \cdot w\,dx$$

where

$$B = \frac{t_D/2 + t_B/2}{h'} = \frac{\frac{1}{2}(t_D + t_B)}{t_B + \frac{1}{2}(t_D + t_S)}$$

When $t_D = t_S$, $B = \frac{1}{2}$ and $T = \frac{1}{2} w\,dx$

The tension energy in a differential element is then

$$dU_T = \frac{T^2}{2K'} = \frac{(Bw\,dx)^2}{2k'\,dx} \quad \text{------------------------(16)}$$

and since

$$-w = h'\frac{dq}{dx} = h'\frac{d^2P}{dx^2}$$

$$dU_T = \frac{B^2 h'^2 \left(\frac{d^2P}{dx^2}\right)^2 dx}{2k'} \quad \text{------------------------(17)}$$

where

k' is the spring constant of the bond, in the normal direction; per inch of length, or

$$k' = \frac{A_B E_B}{t_B} = \frac{W \times 1 \times E_B}{t_B} = \frac{W \times 2(1+\nu)G}{t_B}$$

where

W = width of the bond

G = shearing modulus of elasticity of bond

ν = Poisson's ration

Adding the term, (17) to those in Equation (1)

$$dU = \left[\frac{P^2}{2A_D E_D} + \frac{(Q-P)^2}{2A_S E_S} + \frac{1}{2k}\left(\frac{dP}{dx}\right)^2 + \frac{B^2 h'^2}{2k'}\left(\frac{d^2P}{dx^2}\right)^2\right] dx \text{-----(18)}$$

and

$$U = \int_0^L \left[\frac{P^2}{2A_D E_D} + \frac{(Q-P)^2}{2A_S E_S} + \frac{1}{2k}\left(\frac{dP}{dx}\right)^2 + \frac{B^2 h'^2}{2k'}\left(\frac{d^2P}{dx^2}\right)^2\right] dx \text{-----(19)}$$

In this case the bracketed expression, F, is a function of P, dP/dx and also d^2P/dx^2. Hence, the "extended" form of Eulers Equation must be used. This is (compare to Equation (4))

$$\frac{\partial F}{\partial P} - \frac{d}{dx}\left(\frac{\partial F}{\partial \frac{dP}{dx}}\right) + \frac{d^2}{dx^2}\left(\frac{\partial F}{\partial \frac{d^2P}{dx^2}}\right) = 0 \text{ -----(20)}$$

The higher order term in (20) is obtained by differentiating F as indicated.

$$\frac{\partial F}{\partial \frac{d^2P}{dx^2}} = \frac{B^2 h'^2}{k'}\frac{d^2P}{dx^2}$$

and then

$$\frac{d^2}{dx^2}\left(\frac{\partial F}{\partial \frac{d^2P}{dx^2}}\right) = \frac{B^2 h'^2}{k'}\frac{d^4P}{dx^4}$$

Then, substituting this into Eq. (20) along with the other terms (as in Equation (5)),

$$P\left(\frac{1}{A_S E_S} + \frac{1}{A_D E_D}\right) - \frac{Q}{A_S E_S} - \frac{1}{k}\frac{d^2P}{dx^2} + \frac{B^2 h'^2}{k'}\frac{d^4P}{dx^4} \text{ -----(21)}$$

And, rearranging terms,

$$\frac{d^4P}{dx^4} - \frac{k'}{B^2 h'^2 k}\frac{d^2P}{dx^2} + \frac{k'}{B^2 h'^2}\left(\frac{1}{A_S E_S} + \frac{1}{A_D E_D}\right)P = \frac{4k'Q}{R^2 h'^2 A_S E_S}$$

or

$$\frac{d^4P}{dx^4} - L'\frac{d^2P}{dx^2} + M'P = -N \text{ --------(22)}$$

Where

$$L' = \frac{k'}{B^2 h'^2 k}, \quad M' = \frac{k'}{B^2 h'^2}\left(\frac{1}{A_S E_S} + \frac{1}{A_D E_D}\right), \quad N' = \frac{k'Q}{B^2 h'^2 A_S E_S}$$

Comparing (22) to (6) it is seen that there is now a fourth order term, which considerably complicates the solution, and that the constants are now affected by the stiffness of the bond in the normal direction. The solution of (22) is

$$P = C_1 e^{D_1 x} + C_2 e^{D_2 x} + C_3 e^{D_3 x} + C_4 e^{D_4 x} + \frac{N'}{M'} \quad \text{--(23)}$$

where

$$D_1 = \left(\frac{L' + \sqrt{L'^2 - 4M'}}{2}\right)^{\frac{1}{2}} \; ; \quad D_2 = -D_1$$

$$D_3 = \left(\frac{L' - \sqrt{L'^2 - 4M'}}{2}\right)^{\frac{1}{2}} \; , \quad D_4 = -D_3$$

Although general formulas cannot be written as in the previous (elementary) cases, for any specific problem L', M' and N' and hence D_1-- D_4 are known. Thus, for a specific problem, a solution for P can be obtained from (23). The expressions for q and w will then also be available (by successive differentiation of Eq (23)) as

$$q = D_1 C_1 e^{D_1 x} + D_2 C_2 e^{D_2 x} + D_3 C_3 e^{D_3 x} + D_4 C_4 e^{D_4 x} \quad \text{---(24)}$$

$$w = h'\left[D_1^2 C_1 e^{D_1 x} + D_2^2 C_2 e^{D_2 x} + D_3^2 C_3 e^{D_3 x} + D_4^2 C_4 e^{D_4 x}\right] \quad \text{---(25)}$$

Since there **are** 4 constants, C, 4 boundary conditions are required to define them. For the splice these are

@ X = 0, P = 0 ; @ x = 0, q = 0

@ X = L, P = Q ; @ x = L, q = 0

or, for a symmetrical configuration

$$\text{at } x = \frac{L}{2}, \; P = \frac{Q}{2} \quad \text{and at } x = \frac{L}{2}, \; \frac{dq}{dx} = 0$$

The use of these relationships is illustrated in the following example.

Example:

Determine the values of P, q and w for the sandwich type splice shown in Figure II.7a and consider the normal forces in the bond. The results will also apply to a single lap splice for the assumptions of Art. II.1, that bending is prevented.

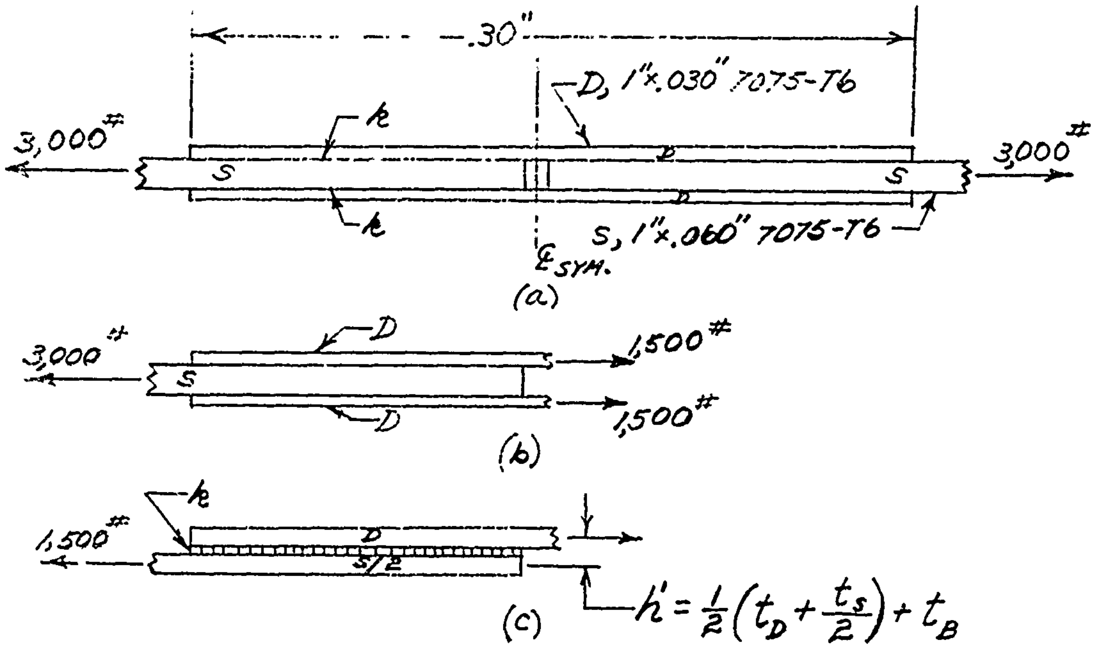


Figure II.7 Idealization of a Splice Structure for Analysis

The splice of (a) is converted to the equivalent structure of (c) for analysis. The following values are assumed for the structure:

$$A_D E_D = \frac{A_S E_S}{2} = .030 \times 10^7$$

Bond is "Redux", having

$$t_B = .0053'', \quad G_B = 100{,}000, \quad W = 1.0'', \quad E = 260{,}000$$

hence

$$k = \frac{WG}{t_B} = 1.87 \times 10^8 ; \qquad k' = \frac{WE}{t_B} = 4.86 \times 10^8$$

For these specific values a solution is obtained as follows:

$$h' = .030 + .0053 = .0353'' \text{ and } B = 1/2$$

Then, $L' = 8340$, $M' = .2595 \times 10^6$, $N' = 194.8 \times 10^6$

and $D_1 = 70.8$, $D_2 = 70.8$, $D_3 = 57.7$, $D_4 = -57.7$

These values and the end conditions result in the final equations (for this particular structure).

$$P = 2.78\times10^{-3} e^{70.8x} + 3.30\times10^{3} e^{-70.8x} - 1.961\times10^{-2} e^{57.7x} - 4.053\times10^{3} e^{-57.7x} + 750$$

$$q = .1966\, e^{70.8x} - 233{,}500\, e^{-70.8x} - 1.132\, e^{57.7x} + 233{,}500\, e^{-57.7x}$$

$$\frac{dq}{dx} = 13.93\, e^{70.8x} + 16{,}530{,}000\, e^{-70.8x} - 65.30\, e^{57.7x} - 13{,}480{,}000\, e^{-57.7x}$$

$$w = .492\, e^{70.8x} + 584{,}000\, e^{-70.8x} - 2.305\, e^{57.7x} - 476{,}000\, e^{-57.7x}$$

From these equations values of the shear stress, f_s (= q/1"), and the tension stress f_t (= w/1") in the bond are calculated at various values of x. The ratios f_s/F_t and f_t/F_t are then computed and plotted in Figure II.8. F_t is the tensile stress in the members away from the joint. The large tension stress in the bond at the ends is of the same order of magnitude as that predicted for similar splices in the "exact" analysis of Reference (3).

The main purpose of this analysis and example is to illustrate that even this most simple additional refinement of the elementary theory results in an analysis effort that is too cumbersome for practical design purposes. The particular refinement illustrated could apply to a glued splice but not to a mechanically fastened one. This is because the fasteners are discrete, they carry bending as well as tension in transferring the shear, they may be "pre-loaded", their spring constants usually vary with the load level, and these effects are partially included in the elementary analysis in using an experimentally obtained spring constant, k, for them. Hence, the elementary analysis, later substantiated by test results, appears to be the only practical one for the case of mechanically fastened joints.

II.5 ANALYSIS OF BONDED JOINT USING THE "GENERALIZED FORCE METHOD"

The previous example was also solved by digital computer using the conventional "Generalized Force Method" for obtaining internal loads in a structure (based on the minimum energy principle). That is, the splice was analyzed as shown in Figure II.9, the equivalent structure for analysis being taken as in (b)

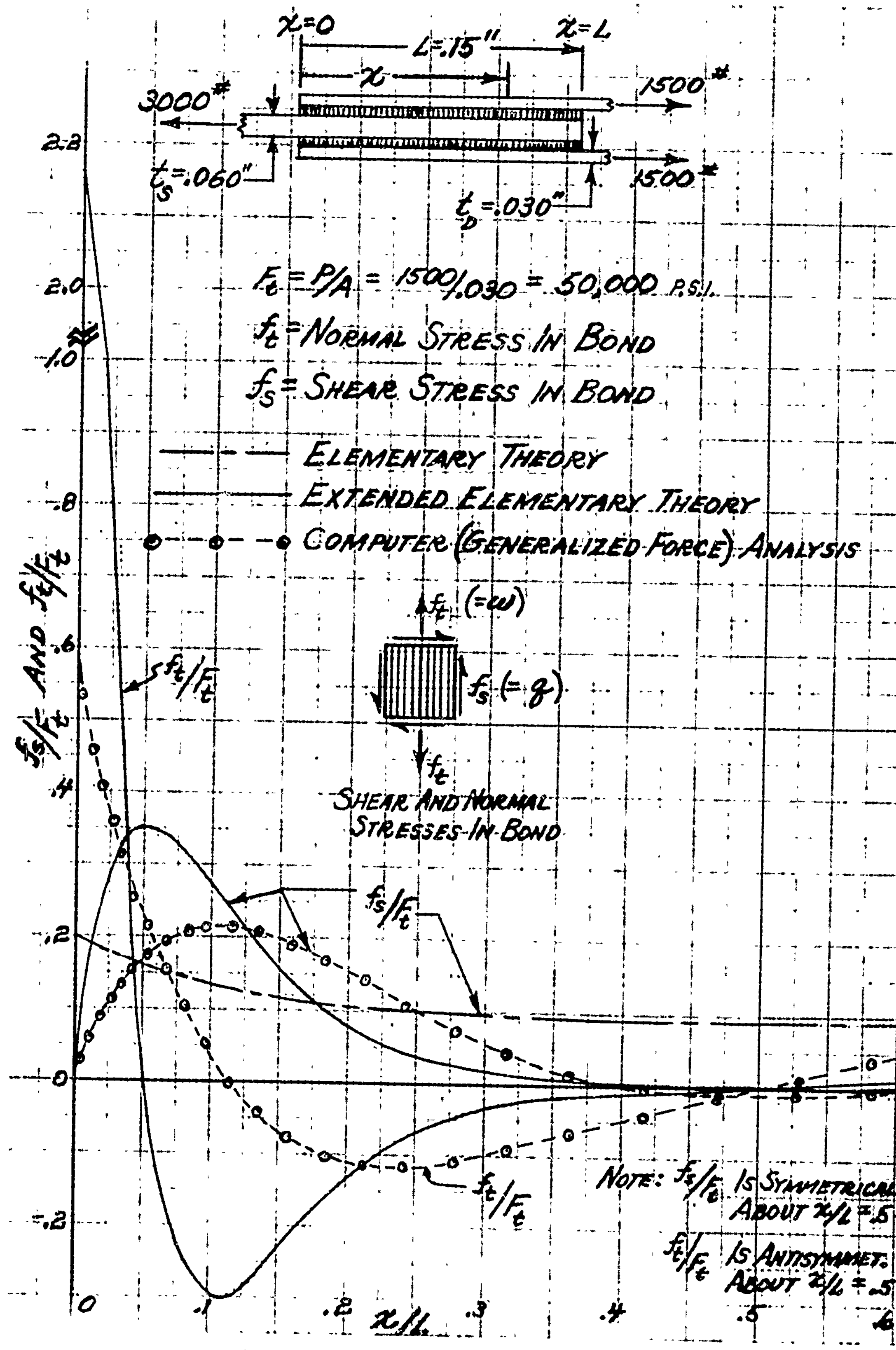


Figure II.8. Internal Stresses In A Bonded Splice

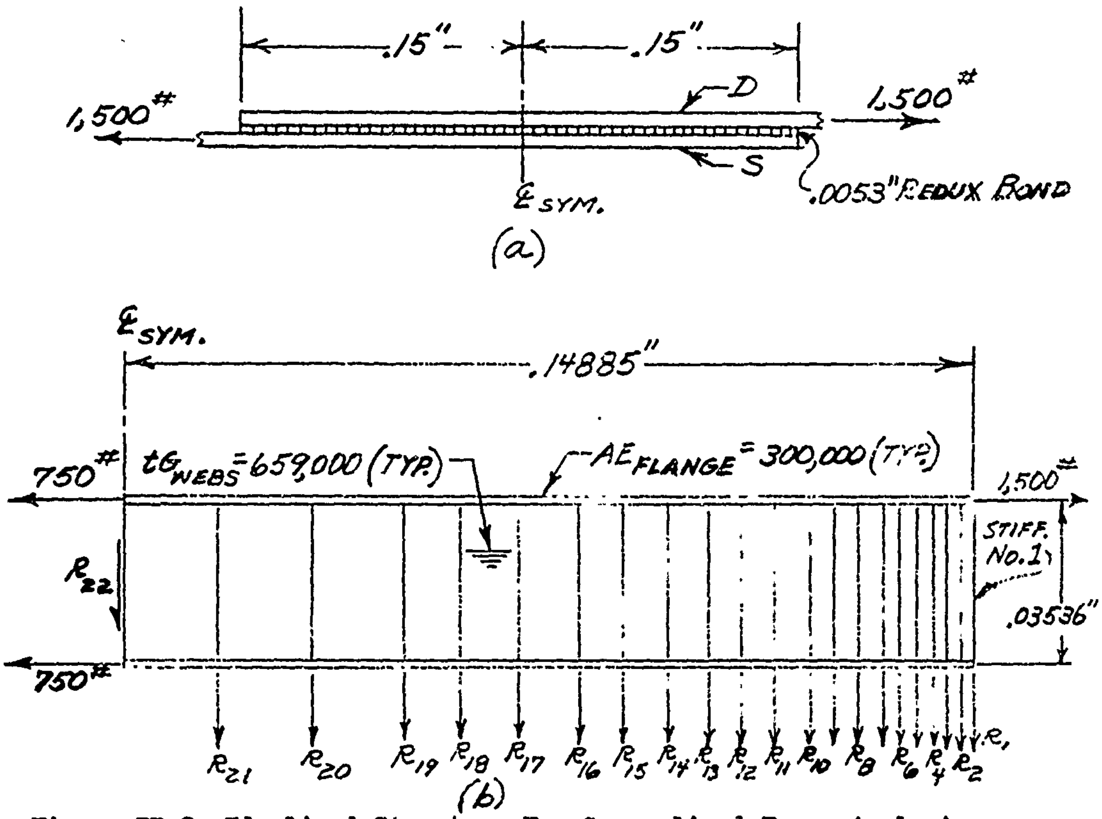


Figure II.9. Idealized Structure For Generalized Force Analysis

TABLE II.1

STIFFENER AE VALUES

NO.	AE	NO.	AE	NO.	AE
1	5250	8	8900	15	18280
2	5250	9	8900	16	25100
3	5250	10	13470	17	25100
4	5480	11	13470	18	25100
5	5480	12	13470	19	37700
6	5480	13	18280	20	37700
7	8900	14	18280	21	37700

The bond was converted into the shear web and stiffeners shown by first dividing it into seven parts of increasing length from the end. Each part was then replaced by three stiffeners (and a web) which would have the same strain energy due to the Reaction loads as would the actual bond. These stiffener AE values are shown in Table II.1.

There were, thus, 22 reactions including the web shear at the centerline of symmetry. The web has a value for tG that provides the same shear rigidity as does the bond. The results (the stiffener loads and web shear flows) are shown in Table II.2.

TABLE II.2

LOADS IN "STIFFENERS" AND SHEAR FLOW IN "WEBS"

STIFF-ENER	LOCATION	LOAD	SHEAR FLOW	STIFF-ENER	LOCATION	LOAD	SHEAR FLOW
n	x in.	R lbs.	q lbs./in	n	x in	R lbs.	q lbs./in.
1	.00115	-59.10	1671	12	.04055	11.95	10438
2	.00345	-52.92	3168	13	.04750	30.24	9583
3	.00575	-47.09	4500	14	.05150	40.12	8448
4	.00810	-43.29	5724	15	.06350	44.96	7177
5	.01500	-37.74	6791	16	.07300	62.23	5417
6	.01290	-30.76	7661	17	.08400	58.38	3766
7	.01605	-42.42	8861	18	.09500	50.10	2349
8	.01995	-31.03	9738	19	.10875	56.34	757
9	.02385	-21.06	10334	20	.12525	33.03	-179
10	.02875	-15.68	10777	21	.14175	10.79	-484
11	.03465	.05	10776	R_{22} = - 484 #/in.			

The results are also plotted in Fig. II.8 as the dashed lines. It is seen that the tension stresses at the end are not as large as the peak values obtained analytically. The maximum shear stress is also lower, but the distributions of shear and tension stresses are of similar form. Possibly using more elements in the computer solution would have given better agreement in this respect, but this was not investigated further. The reactions conform to the basic assumptions of restraint against bending; thus, these analyses would be more representative of a sandwich type splice, than for a lap splice, in actual practice.

An extended digital computer analysis of this type might be useful in analyzing the more complicated splices involving composite structural materials. Since such materials consist of multi-layers, any purely analytical effort would become too cumbersome for practical application and the numerous possible configurations would require too massive an amount of data for a purely empirical approach. (The simple elementary theory is inadequate since it does not account for the high tension stresses at the <u>ends</u> of the layers.)

II.6 SUMMARY OF FORMULAS

This article presents a summary of theoretical formulas for various doubler and splice structural configurations. These have been generated as illustrated in Article II.2 and II.3 and are subject to the same assumptions and limitations as discussed earlier in using the elementary theory. In all cases illustrated the formula for P gives the load in the upper member, D. The load in S can then be obtained from statics.

The designer would usually be interested in only 2 results in using these formulas, namely:

a. The maximum (end) fastener load, which will be that developed over a distance, p, from the end ($x = p$) in either the case of a doubler or splice.*

b. The load developed in the doubler, at the station $x = L/2$.

Hence the practical useage of the formulas is not as laborious as their form would indicate.

The formulas can, of course, also be used to obtain "rough estimates" of loads and shear flows in non-uniform (i.e., tapered or stepped) members. This would be done by substituting "average" values for A, E and k. Such members are much more accurately, analyzed, however, as discussed in Section III, using the numerical procedure.

Seven cases are presented. For each case the basic differential equation is shown, for informative purposes only. The equations numbered 1, 2 and 3 are used for load predictions. If desired, hyperbolic functions can be used to replace some of the exponential forms since

$$e^{z} - e^{-z} = 2 \sinh z$$

and

$$e^{z} + e^{-z} = 2 \cosh z$$

This might be more convenient in cases e, f, and g and is illustrated for case g.

* When $t_D \neq t_g$, let $x = p$ be near the end of the thicker member in the splice (i.e., let D be the thicker member).

a.

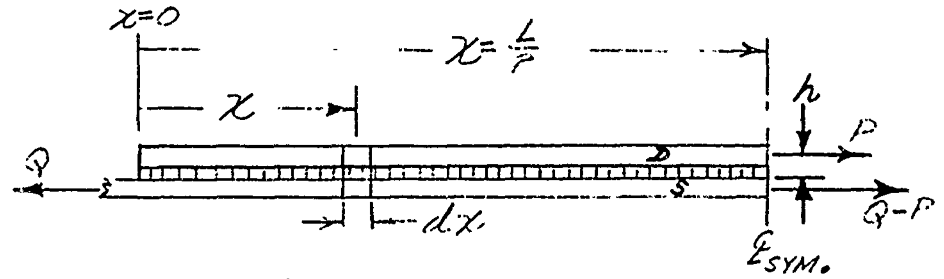


Figure II.10 Half of a Doubler Installation

$$\frac{d^2P}{dx^2} - MP = -N$$

where

1. $P = C_1\left(e^{\sqrt{M}x} + e^{\sqrt{M}L} \cdot e^{\sqrt{M}x}\right) + N/M$
2. $q = \sqrt{M}\, C_1\left(e^{\sqrt{M}x} - e^{\sqrt{M}L} \cdot e^{\sqrt{M}x}\right)$
3. $w = hMC_1\left(e^{\sqrt{M}x} + e^{\sqrt{M}L} \cdot e^{\sqrt{M}x}\right)$

$$\left\{ C_1 = \frac{-N/M}{1+e^{\sqrt{M}L}} \qquad N = \frac{kQ}{A_S E_S} \qquad M = k\left(\frac{1}{A_S E_S} + \frac{1}{A_D E_D}\right) \right\}$$

Approximate Equations:

1.' $P \approx \frac{N}{M}\left(1-e^{-\sqrt{M}x}\right)$ — At $x = L/2$, $P \approx N/M$

2.' $q \approx \frac{N}{\sqrt{M}} e^{-\sqrt{M}x}$ — At $x = 0$, $q \approx N/\sqrt{M}$

3.' $W \approx -hNe^{-\sqrt{M}x}$

b.

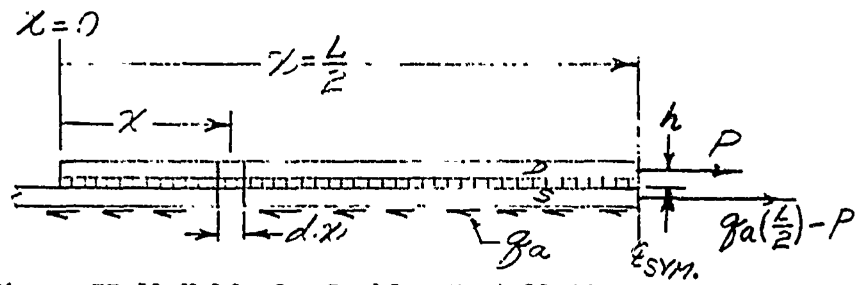


Figure II.11 Half of a Doubler Installation

$$\frac{d^2P}{dx^2} - MP = -N_o x$$

where

1. $P = C_1\left(e^{\sqrt{M}x} - e^{-\sqrt{M}x}\right) + \frac{N_a x}{M}$
2. $q = \sqrt{M}C_1\left(e^{\sqrt{M}x} + e^{-\sqrt{M}x}\right) + \frac{N_a}{M}$
3. $w = hMC_1\left(e^{\sqrt{M}x} - e^{-\sqrt{M}x}\right)$

$$\left\{ C_1 = \frac{-N}{M^{3/2}\left(e^{\sqrt{M}L/2} + e^{-\sqrt{M}L/2}\right)} \qquad N_a = \frac{kq_a}{A_S E_S} \right.$$

where

$$\left. M = k\left(\frac{1}{A_S E_S} + \frac{1}{A_D E_D}\right) \right\}$$

c.

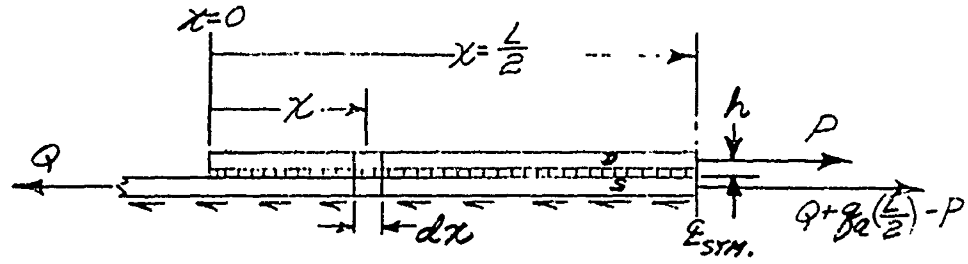


Figure II.12 Half of a Doubler Installation

$$\frac{d^2P}{dx^2} - MP = -(N+N_a x)$$

1. P =
2. q =
3. W =

These are obtained by superposition of the results of separate analyses using cases a and b.

d.

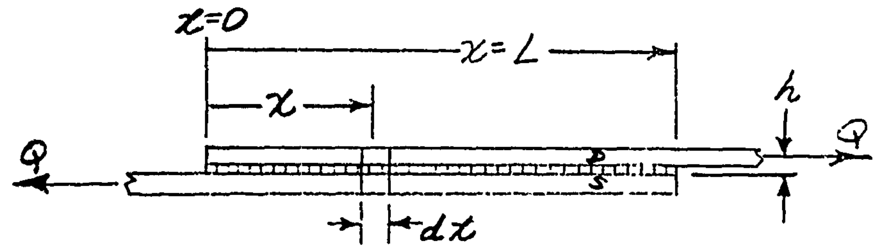


Figure II.13 A Splice Installation

$$\frac{d^2P}{dx^2} - MP = -N$$

where

1. $P = C_1(e^{\sqrt{M}x} - e^{-\sqrt{M}x}) + \frac{N}{M}(1-e^{-\sqrt{M}x})$
2. $q = \sqrt{M}C_1(e^{\sqrt{M}x} + e^{-\sqrt{M}x}) + Ne^{-\sqrt{M}x}$
3. $W = h\left[MC_1(e^{\sqrt{M}x} - e^{-\sqrt{M}x}) - Ne^{-\sqrt{M}x}\right]$

$$\left\{\begin{aligned} C_1 &= \frac{Q - \frac{N}{M}(1-e^{-\sqrt{M}L})}{e^{\sqrt{M}L} - e^{-\sqrt{M}L}} \\ N &= \frac{kQ}{A_S E_S} \\ M &= k\left(\frac{1}{A_S E_S} + \frac{1}{A_D E_D}\right) \end{aligned}\right\}$$

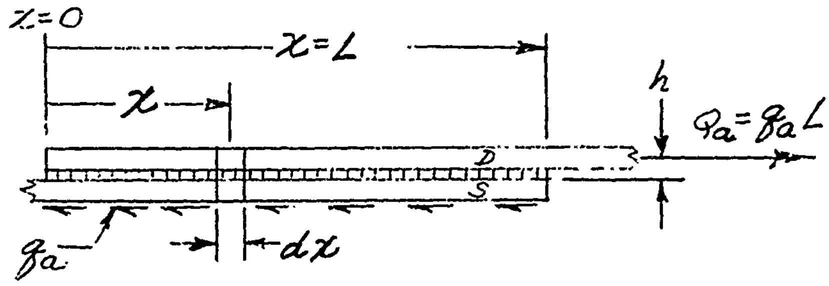
e. 

Figure II.14 A Splice Installation

$$\frac{d^2P}{dx^2} - MP = -N_a x$$

1. $P = (Q_a - \frac{N}{M})\left(\frac{e^{\sqrt{M}x} - e^{-\sqrt{M}x}}{e^{\sqrt{M}L} - e^{-\sqrt{M}L}}\right) + \frac{N_a x}{M}$

2. $q = (Q_a - \frac{N}{M})\sqrt{M}\left(\frac{e^{\sqrt{M}x} + e^{-\sqrt{M}x}}{e^{\sqrt{M}L} - e^{-\sqrt{M}L}}\right) + \frac{N_a}{M}$

3. $w = h\left[(Q_a - \frac{N}{M})\, M \left(\frac{e^{\sqrt{M}x} - e^{-\sqrt{M}x}}{e^{\sqrt{M}L} - e^{-\sqrt{M}L}}\right)\right]$

where

$$\left\{\begin{aligned} N &= \frac{kQ_a}{A_S E_S} \\ N_a &= \frac{kq_a}{A_S E_S} \\ M &= k\left(\frac{1}{A_S E_S} + \frac{1}{A_D E_D}\right) \end{aligned}\right\}$$

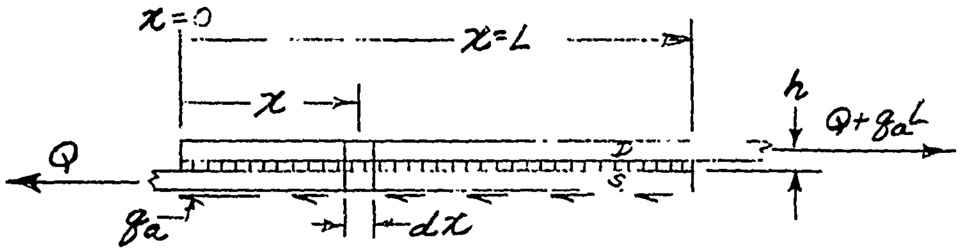
f. 

Figure II.15 A Splice Installation

$$\frac{d^2P}{dx^2} - MP = -(N + N_a x)$$

$\left.\begin{array}{l} 1.\ P = \\ 2.\ q = \\ 3.\ w = \end{array}\right\}$ These are obtained by superposition of the results of separate analyses using cases d and e.

g.

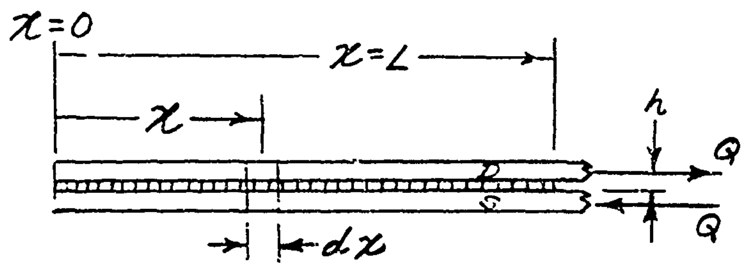


Figure II.16 A Splice Installation

$$\frac{d^2P}{dx^2} - MP = 0$$

Where

1. $P = Q \left(\frac{e^{\sqrt{M}x} - e^{-\sqrt{M}x}}{e^{\sqrt{M}L} - e^{-\sqrt{M}L}}\right) = Q \frac{\text{Sinh}\sqrt{M}x}{\text{Sinh}\sqrt{M}L}$ $\left\{ M = k\left(\frac{1}{A_S E_S} + \frac{1}{A_D E_D}\right) \right\}$

2. $q = \sqrt{M}Q \left(\frac{e^{\sqrt{M}x} + e^{-\sqrt{M}Z}}{e^{\sqrt{M}L} - e^{-\sqrt{M}L}}\right) = \sqrt{M}Q \frac{\cosh\sqrt{M}x}{\sinh\sqrt{M}L}$

3. $W = hMQ \left(\frac{e^{\sqrt{M}x} - e^{-\sqrt{M}x}}{e^{\sqrt{M}L} - e^{-\sqrt{M}L}}\right) = hMQ \frac{\sinh\sqrt{M}x}{\sinh\sqrt{M}L}$

EXAMPLE PROBLEM

A doubler installation is shown in Figure II.17. This is the same structure as in Figure III.4 without the slop at the left end fastener. Determine

a) The shear load developed in the end fasteners

b) The load developed at the center of the doubler and of the base structures

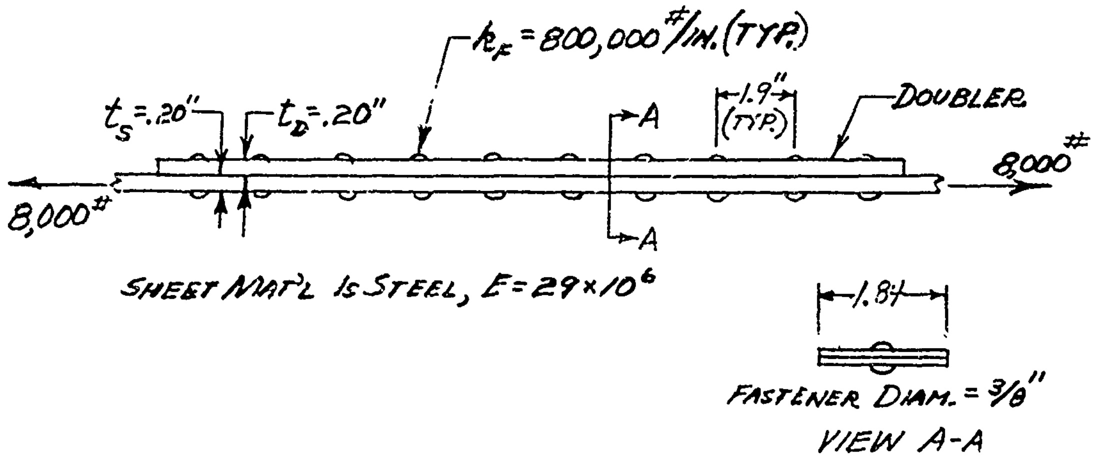


Figure II.17. A Doubler Installation

This is representative of Case a. The "approximate" equations will be used. The various constants are

$$k = \frac{k_F}{p} = \frac{800{,}000}{1.9} = \underline{421{,}000}\ \#/\text{In}/\text{In}$$

$$A_S = \text{NET EFFECTIVE AREA}^* = (\text{Width} - .8D)\ t_S$$

$$= 1.84 - .8\ (.375)\quad (.20) = \underline{.308}\ \text{in}^2$$

$$A_D = \left[1.84 - .8\ (.375)\right]\ (.20) = \underline{.308\ \text{in}^2}$$

$$N = \frac{kQ}{A_S E_S} = \frac{421{,}000\ (8000)}{.308\ (29\text{x}10^6)} = \underline{377}$$

$$M = k\left(\frac{1}{A_S E_S} + \frac{1}{A_D E_D}\right) = 421{,}000\left[\frac{1}{.308(29\text{x}10^6)} + \frac{1}{.308(29\text{x}10^6)}\right] = \underline{.0943}$$

$$\sqrt{M} = \sqrt{.0943} = \underline{.307}$$

* See Figure V.4

a) The load at the left end fastener is calculated using formula 1' of case a as $P_{F_1} = P_{D_{x=p}} - P_{D_{x=0}} = P_{D_{x=p}}$ hence,

$$P \approx \frac{N}{M}\left(1 - e^{-\sqrt{M}x}\right) = \frac{377}{.0943}\left(1 - e^{-.307 \times 1.9}\right) = \frac{377}{.0943}\left(1 - \frac{1}{1.795}\right) = \underline{\underline{1770^{\#}}}$$

That is, since each fastener has been replaced by a bond 1.9" long the load developed over this length of bond is the fastener load. Due to symmetry the load on the right end fastener is the same as that on the left end fastener.

b) The load developed at the center of the doubler, $(x = \frac{L}{2})$ is

$$P \approx \frac{N}{M}\left(1 - e^{-\sqrt{M}x}\right) = \frac{377}{.0943}\left(1 - e^{-.307 \times 9.5}\right) = \underline{\underline{3780^{\#}}}$$

The load in the base structure is then, from statics,

$$P_s = Q - P = 8000 - 3780 = \underline{\underline{4220^{\#}}}$$

EXAMPLE PROBLEM

A splice is shown in Figure II.18. This is the same splice as in Figure III.4 without the "slop" at the left end fastener. Determine

a) The shear load developed in the end fasteners

b) The load in the center elements of the splice member (at x = L/2)

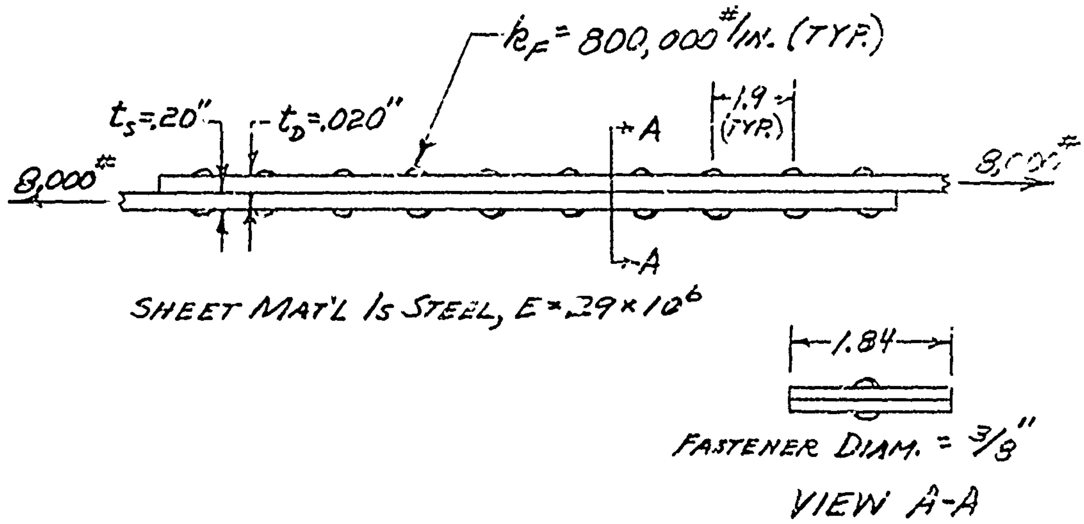


Figure II.18. A Splice Installation

This is Case d, and, as in the previous examples,

$k = 421{,}000\#/\text{In}, \ A_s = A_D = .308 \ \text{In}^2,$

$N = 377, \quad M = .0943, \quad \sqrt{M} = .307$

And, for this case,

$$C_1 = \frac{Q - \frac{N}{M}\left(1 - e^{-\sqrt{M}L}\right)}{e^{\sqrt{M}L} - e^{-\sqrt{M}L}} = \frac{8000 - \frac{377}{.0943}\left(1 - e^{-.307\times 19.0}\right)}{e^{.307\times 19.0} - e^{-.307\times 19.0}} = \frac{8000-3990}{340 - \frac{1}{340}} = \underline{\underline{11.8}}$$

a) the load in the (left) end fastener is determined as that developed over the end (1.9") segment of the bond, as in the previous example problem.

$$P_{F_1} = C_1\left(e^{\sqrt{M}x} - e^{-\sqrt{M}x}\right) + \frac{N}{M}\left(1 - e^{-\sqrt{M}x}\right)$$

$$= 11.8\left(e^{.307\times 1.9} - e^{-.307\times 1.9}\right) + \frac{377}{.0943}\left(1 - e^{-.307\times 1.9}\right)$$

$$= 11.8\left(1.793 - \frac{1}{1.793}\right) + 3998\left(1 - \frac{1}{1.793}\right)$$

$$= \underline{\underline{1792^{\#}}}$$

Since the members D & S have the same values of AE (or since $t_s = t_D$) the right end fastener will feel the same load. If $A_D E_D \neq A_s E_s$, the end fasteners will not feel the same load. The largest load will be at the end of the stiffer member.

b) The load developed in the center segment of the upper member (D) is determined from Eq. d.1, for x = L/2 = 9.5",

$$P = 11.8\left(e^{.307\times 9.5} - e^{-.307\times 9.5}\right) + \frac{377}{.0943}\left(1 - e^{-.307\times 9.5}\right)$$

$$= 218 + 3782$$

$$= \underline{\underline{4000^{\#}}}$$

The load in the center segment of the lower splice member(S) is then, from statics,

$$P_S = Q - P = 8000 - 4000 = \underline{\underline{4000}}$$

Had the members D and S not had the same value of AE, (or $t_S \neq t_D$) the loads P_S and P_D would not have been equal at the center segment.

These two examples are also solved by the numerical method in Section III, assuming one of the end fasteners to be installed in a "sloppy" (oversize) hole.

SECTION III

METHOD 2 - NUMERICAL METHOD FOR HAND ANALYSES

III.1 INTRODUCTION

The previous analytic equations apply only to the particular case involving uniform members. In general the geometry and the attachments will vary along the length. Hence, the Constants M and N of Eq. (6) will be functions of x and simple solutions will not be available. In this case a numerical integration of the differential equation (6), for each specific problem would be required. This could, of course, be done and used as a tool (but not for an accurate final load distribution) in an analysis of an actual glued joint. However, in the case of discrete fasteners it is advantageous to use a different procedure, which allows for including the effects of fastener-hole clearance ("slop") and plasticity. In addition, it is also more meaningful to the engineer.

III.2 NUMERICAL ANALYSIS METHOD FOR DOUBLER INSTALLATIONS

A practical engineering method for determining the distribution of fastener loads in a doubler or splice by hand analysis is often helpful. Such a procedure is described below, first for the case of a doubler. It is essentially one of successive trials using the principle of static equilibrium as the criteria for the correct distribution of internal loads. Figure III.1 shows a base structure, S, subjected to the applied loadings Q_L, Q_R, and q_a, q_a being an applied shear flow. A reinforcing member, or doubler, D, is attached to S by the mechanical fasteners, F. The "gap" between D and S is exaggerated for purposes of illustration.

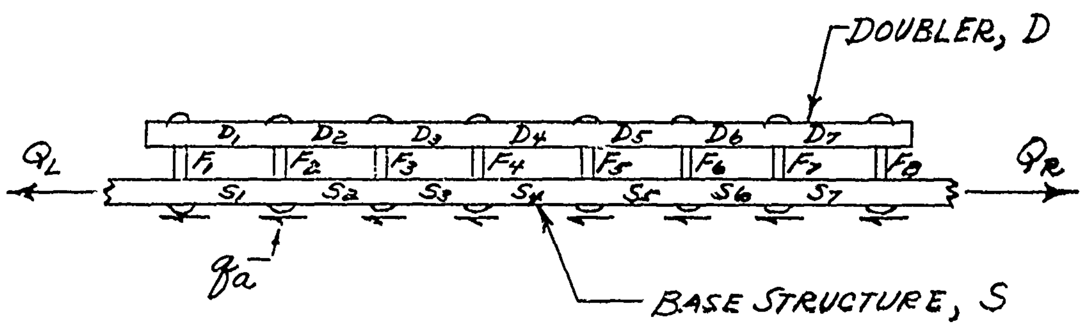


Figure III.1 A Doubler Installation

As the member S stretches under the applied loads, the common fasteners will, in turn, tend to stretch the member D. Loads will thus be generated in the fasteners. Considering only those forces in the axial direction, the shear loads in the fasteners can be determined as follows. Letting the end fastener, #1, at the base structure be the reference point for axial stretching, or displacement, the resulting relative movement is as shown in Figure III.2. The dotted lines show the displaced positions.

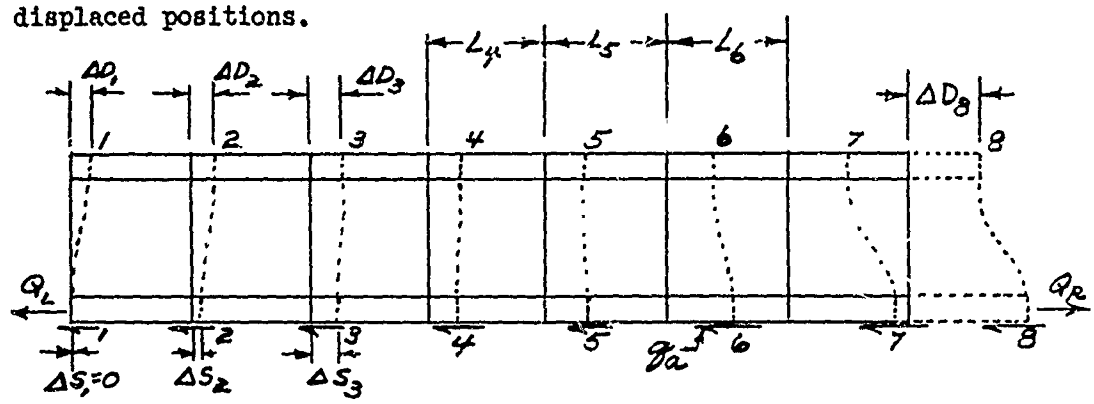


Figure III.2 Displacement of Members Due to Applied Loads

Figure III.3 shows the applied and the internal loads and also the sign convention used. That is, all applied and internal loads are positive when acting as shown.

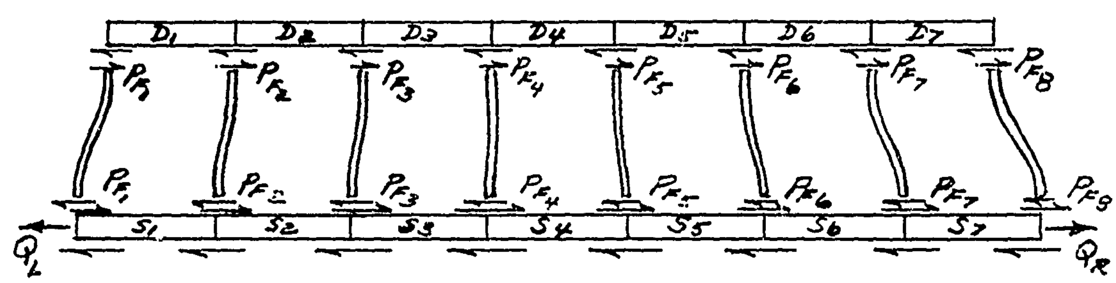


Figure III.3 Sign Convention for Applied Loads and Internal Loads

As in Figure III.2 let ΔD_n be the total movement of each fastener at the doubler and ΔS_n be that at the base structure. Then, at the doubler,

$\Delta D_1 = \delta_{F_1} = \Delta\delta_{F_1}$ = the displacement at the first fastener, #1, which is also the net strain (in shear) for fastener #1, $\Delta\delta_{F_1}$, since $\Delta S_1 = 0$.

$$\Delta D_2 = \delta_{F_1}$$ + the total strain, or stretch, in the doubler element 1.

Then, in general, at any point, n,

$$\Delta D_n = \delta_{F_1} + \sum^{n-1} \delta_{D_n}$$

The total displacement at any fastener on the base structure, S, will be the sum of the individual total strains of the elements, S_n, up to that point, or,

$$\Delta S_n = \sum^{n-1} \delta_{S_n}$$

The <u>net</u> strain (in shear) of any fastener will, therefore, be the <u>difference</u> between the total displacements of its ends, at D and at S. This is

$$\Delta \delta_{F_n} = \Delta D_n - \Delta S_n$$

$$= \delta_{F_1} + \sum^{n-1} \delta_{D_n} - \sum^{n-1} \delta_{S_n} \quad \text{----------(26)}$$

The corresponding fastener load can then be determined from the relationship

$$P_{F_n} = k_{F_n} \times \Delta \delta_{F_n} \quad \text{---------------(27)}$$

where k_F = spring constant of the fastener-sheet combination, discussed further in Section V.

Once P_{F_n} is known the corresponding loads in the next axial elements P_{D_n} and P_{S_n} are defined, since as indicated in Figure III.3,

$$P_{D_n} = \sum^{n} P_{F_n} \quad \text{---------------(28)}$$

and

$$P_{S_n} = Q_L + \sum^{n} q_a \left(\frac{L_{n-1} + L_n}{2} \right) - \sum^{n} P_{F_n} \quad \text{------(29)}$$

where L_n = length of elements S (or D) with $L_0 = 0$ (i.e., for n = 1)

The total axial strain in the elements S_n and D_n can then be calculated as

$$\delta_{D_n} = P_{D_n}/k_{D_n} \text{ ---------------------------------------(30)}$$

and

$$\delta_{s_n} = P_{s_n}/k_{s_n} \text{ ---------------------------------------(31)}$$

Where k_n = the spring constants of the elements D_n and S_n (i.e., AE/L), as discussed in Section V.

The next fastener load, $P_{F_{n+1}}$, can then be calculated from Equations (26) and (27) and then all those remaining in a similar successive repetitive manner.

An engineering procedure for determining the fastener loads is therefore as follows:

a. Assume a value for the first fastener load P_{F_1} and using Eq. (27) calculate the corresponding fastener strain, δ_{F_1}. (This assumption is discussed later)

b. Calculate the strains in the members S_1 and D_1 from Eq. (30) and (31).

c. Calculate the strain in the second fastener, $\Delta\delta_{F_2}$, using Eq. (26) and then calculate the fastener load, P_{F_2} using Eq. (27).

d. Repeat steps (b) and (c) repetitively until all of the fastener loads have been determined.

e. Add up all of the fastener loads. If their sum is not zero (needed for static balance of the doubler, as in Figure III.3) the initial guess in step a is in error. Then assume another value in step a and repeat the procedure. After a few trials the true distribution of fastener loads can be determined, with sufficient accuracy for engineering purposes. Plotting the values of each assumed fastener load versus the corresponding error in static balance (i.e., versus the sum of the fastener loads) will assist in rapidly determining the true initial fastener load.

If there is present a clearance, or "slop", at any fastener and hole, the effect can be accounted for by modifying Equation (26). That is, the fastener will not be strained through the full relative movement, $\Delta D_n - \Delta S_n$ since all or part of this will be used in

"closing up" the clearance. Thus, if the fastener hole clearance is denoted by Δc , Equation (26) becomes

$$\Delta\delta_{F_n} = \delta_{F_1} + \sum^{n-1} \delta_{D_n} - \sum^{n-1} \delta_{S_n} - \Delta c_n \text{ ------}(32)$$

However, there is a limit here in that Δc can, at most, only reduce $\Delta\delta_{F_n}$ to zero, as in the case of a large clearance. That is, it cannot load up the fastener in the opposite direction.

The procedure can be carried out by hand most easily if a tabular form is used. Such a tabular form is shown in the following example.

A first guess for the end fastener load can be made, arbitrarily, by first assuming that the doubler will carry a portion of the applied load in proportion to its stiffness. That is

$$P_{DOUBLER} = Q\left(\frac{A_D E_D}{A_D E_D + A_S E_S}\right)$$

It can then be assumed that the outer 25% of the fasteners will pick up this load uniformly. Thus, if there are N fasteners (or rows of fasteners) and Q is the average applied end load, the initial guess for the end fastener load would be

$$P_{F_1} = \frac{Q}{N/4}\left(\frac{A_D E_D}{A_D E_D + A_S E_S}\right) = \frac{4Q}{N}\left(\frac{A_D E_D}{A_D E_D + A_S E_S}\right)$$

where $Q = \dfrac{Q_L + Q_R}{2}$ and $A_D E_D$ and $A_S E_S$ are average values.

The analysis is then carried out using the tabular form. (Table III.1).

The second guess is made in such a manner as to reduce the error (i.e., $\sum P_{F_n}$) that results from carrying out the procedure the first time. That is, if $\sum P_{F_n} > 0$, the second guess would be a smaller load and if $\sum P_{F_n} < 0$, it would be a larger one. The second analysis is then carried out, followed by a third analysis, etc. as necessary.

EXAMPLE PROBLEM:

Determine the internal load distribution in the doubler - sheet structure shown in Figure III.4

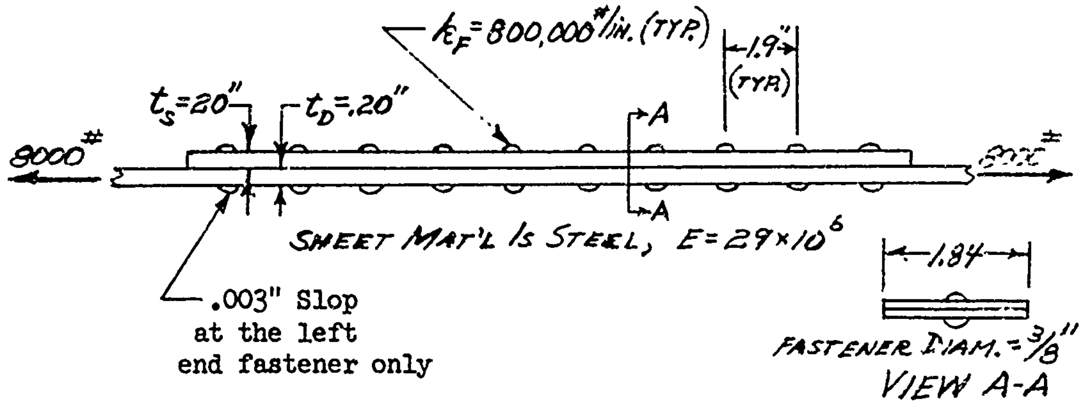


Figure III.4. A Doubler Installation

The Fastener Spring constant is given as

$$k_F = 800,000 \ \#/\text{IN.}$$

The Doubler and Sheet Spring constants are then calculated (as discussed in Art. V.3) as

$$k_D = \frac{A_e E}{L} = \frac{(1.84 - .375 \times .80)(.20)(29)(10)^6}{1.9} = \underline{\underline{4.7 \times 10^6}}$$

$$k_S = \frac{A_e E}{L} = \underline{\underline{4.7 \times 10^6}}$$

These values of k_F, k_D and k_S are then listed in Col. (5), (8) and (14) respectively of Table III.1. The applied load of 8000# is listed in Col. (12) and 0# is listed in (10) since no intermediate loads exist.

An initial value for the first fastener load would be taken as, (if no "slop" were present)

$$P_{F_1} = \frac{Q}{N/4}\left(\frac{A_D E_D}{A_D E_D + A_S E_S}\right) = \frac{8000}{10/4}\left(\frac{8.93 \times 10^6}{8.93 \times 10^6 + 8.93 \times 10^6}\right) = \underline{\underline{1600^{\#}}}$$

but since .003" "slop" is present at this left end fastener this is arbitrarily guessed to be only half as much, or

$$P_{F_1} = 800\#$$

Thus 800# is listed in Col. (6) for n = 1. The first trial Table III.1 is then completed (working "backwards" to obtain the value for Col. (2) for n = 1)

For the correct value of P_{F_1} the doubler load at the last fastener (#10) will be zero, or Col. $(7)_{10} = 0$. Since in this trial $(7)_{10} = 101{,}010 > 0$, another trial is necessary assuming a smaller value for Col. $(6)_1$

After several trials, including plotting the "error" (which is the value in Col. $(7)_{10}$) vs. the assumed value, Col. $(6)_1$, the final loads are obtained. It is seen that $(7)_{10} = 6\# \approx 0$, sufficiently accurate for common engineering purposes.

This relatively simple analysis is all that is necessary for those installations where all internal loads are in the elastic range (i.e., where no yielding is to be allowed, usually at limit load).

If the slop is "too large" at the left end fastener #1, the load in the fastener must of course be zero. This would be indicated in a tabular solution if assuming $P_1 = 0$ was not "small enough" to obtain a static balance ($(7)_{n=N} \neq 0$). Actually, the smallest value of slop that causes the first fastener load to be zero can be obtained as follows. Assume $P_{F_1} = 0$. Then, by "trial and error" tables, find the value of ΔC_1 ($(3)_1$) that gives a static balance. For this and any larger value of slop the first fastener load is zero. That is, the first fastener is "out of action". The true load distribution in the other fasteners is then obtained by starting with fastener #2 (i.e. ignoring fastener #1 since $P_1 = 0$) and assuming a value for fastener #2. Should #2 have too much slop also, then $P_{F_1} = 0$, $P_{F_2} = 0$ and the distribution of loads must be obtained by "starting" with fastener #3, etc.

TABLE III.1

TABULAR METHOD FOR DOUBLER ANALYSIS, ACCOUNTING FOR APPLIED AXIAL END OR INTERMEDIATE (SHEAR FLOWS) LOADS AND ATTACHMENT "SLOP"

1	2	3	4 *	5	6	7	8	9	10 **	11	12	13	14	15	16	
FASTENER OR STATION	Δ DIFF IN STRAIN	FASTENER "SLOP"	FASTENER STRAIN (SHEAR)	FASTENER SPRING CONST.	FASTENER LOAD	DOUBL' LOAD	DOUBLER SPRING CONST.	DOUBLER STRAIN	INTERM. LOADS	ACCUM. INTERM. LOADS	ACCUM. APPLIED LOADS	LOAD IN BASE STRUCT.	BASE STRUCT. SPR. CON.	BASE STRUCT STRAIN	DIFF. IN STRAIN	
	$\Delta(\delta_S-\delta_D)$	Δc	$\Delta\delta_F$	k_F	P_F	P_D	k_D	δ_D	$q_a \times L$			P_S	k_S	δ_S	$\delta_S-\delta_D$	
n	$(2)_{n-1}-(16)_{n-1}$	GIVEN	$(2) \pm (3)$ *	GIVEN	$(4)\times(5)$	$\sum (6)$	GIVEN	$(7)/(8)$	GIVEN	$\sum (10)$	$Q_L + (11)$	$(12)-(7)$	GIVEN	$(13)/(14)$	$(15)-(9)$	
	$\times 10^6$	$\times 10^6$	$\times 10^6$	$\times 10^{-6}$			$\times 10^{-6}$	$\times 10^6$					$\times 10^{-6}$	$\times 10^6$	$\times 10^6$	
1	4000	3000	1000	.80	800	800	4.7	170	0	0	8000	7200	4.70	1532	1362	FIRST TRIAL
2	2638	0	2638	"	2110	2910	"	619	"	"	"	5090	"	1184	565	
3	2073	"	2073	"	1650	4568	"	973	"	"	"	3432	"	729	-244	
4	2317	"	2317	"	1852	6420	"	1365	"	"	"	1580	"	336	-1029	
5	3346	"	3346	"	2680	9100	"	1934	"	"	"	-1100	"	-234	-2168	
6	5514	"	5514	"	4410	13510	"	2870	"	"	"	-5510	"	-1170	-4040	
7	9554	"	9554	"	7640	21150	"	4500	"	"	"	-13150	"	-2790	-7290	
8	16844	"	16844	"	13460	34610	"	7360	"	"	"	-26610	"	-5660	-13020	
9	29864	"	29864	"	23900	58510	"	12440	"	"	"	-50510	"	-10740	-23180	
10	53044	"	53044	"	42500	101010	"	--	"	"	"	--	"	--	--	
1	3482	3000	482	.80	385	385	4.7	82	0	0	8000	7615	4.70	1620	1538	FINAL TRIAL
2	1944	"	1944	"	1553	1938	"	413	"	"	"	6062	"	1290	877	
3	1067	"	1067	"	854	2792	"	594	"	"	"	5208	"	1108	514	
4	553	"	553	"	442	3234	"	688	"	"	"	4766	"	1014	326	
5	227	"	227	"	182	3416	"	727	"	"	"	4584	"	977	250	
6	-23	"	-23	"	-18	3398	"	724	"	"	"	4602	"	980	256	
7	-279	"	-279	"	-222	3176	"	677	"	"	"	4824	"	1026	349	
8	-628	"	-628	"	-501	2675	"	569	"	"	"	5325	"	1133	564	
9	-1192	"	-1192	"	-953	1722	"	367	"	"	"	6278	"	1336	969	
10	-2161	"	-2161	"	-1728	-6	--	--	--	--	--	--	--	--	--	

* (3) Makes (4) less than (2), but cannot reverse the "sign". (i.e., can "reduce" (2) only to zero).

** The intermediate load is due to either an applied shear flow, qa, or a local applied axial load. Assume value for $(6)_{n=1}$ & complete table. For correctly assumed $(6)_{n=1}$, $(7)_{n=N} = 0$

III.3 NUMERICAL METHOD FOR SPLICES

In the case of a splice the same general procedure would be used as can be seen from an inspection of Figure III.5 compared to Figure III.2. In this case, however, there is an applied load acting on each member, S and D. Thus, the criteria for the correct fastener load distribution will be, from statics,

$$\sum^{n} P_{F_n} = \text{Applied Loads on either member.}$$

This can be seen in Figure III.6 which shows the applied and internal loads for a splice configuration. As discussed in Section II a sandwich type splice is converted to a single lap arrangement for purposes of analysis.

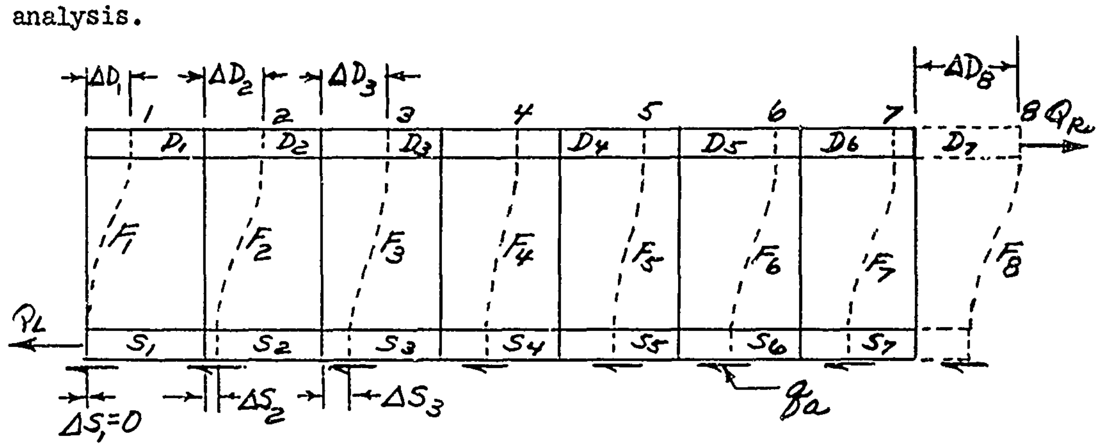


Figure III.5. Displacement of Members Due to Applied Loads

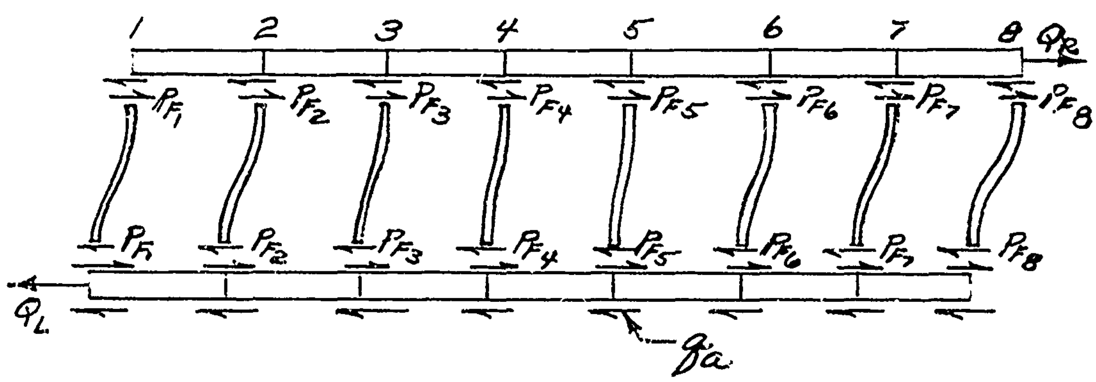


Figure III.6. Sign Convention For Applied Loads and Internal Loads

In general the end fastener loads will be largest and those in the middle the smallest. The procedure can be carried out in tabular form as discussed previously by assuming a value for P_{F1}, the first fastener load. A value for the first guess can be taken as,

$$P_{F_1} \approx \frac{2Q}{N}$$

which is obtained by assuming that 1/2 of the average applied end load is transferred by the outer 25% of the fasteners at each end. The following example illustrates the method for the case of a splice.

EXAMPLE PROBLEM.

Determine the internal load distribution in the splice structure shown in Figure III.7

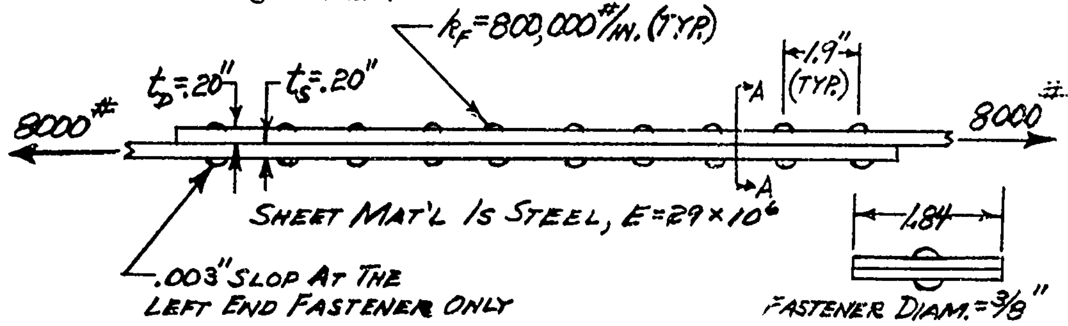


Figure III.7. A Splice Installation

The fastener spring constants are given as 800,000 #/in. The doubler and base structure spring constants are computed as in the previous example (and have the same values).

These values and the applied load of 8000# are listed in Table III.2, as discussed for Table III.1.

An initial guess for the first fastener load, Col. ⑥$_1$, would be taken as (if no "slop" were present)

$$P_{F_1} = \frac{2Q}{N} = \frac{2(8000)}{10} = \underline{\underline{1600}}^{\#}$$

But, since .003" slop is present at this left end fastener this load is arbitrarily guessed to be only half as much or

$$P_{F1} = 800\#$$

The trials are then carried out in Table III.2 as discussed for Table III.1. However, in this case, a splice, the correct value for ⑥$_1$ results in the value of ⑦$_{10}$ being equal to the applied end load of 8,000# (instead of zero, as for the doubler).

In this case, a splice, the "error" would be

$$\text{Error} = \text{Col. ⑦}_{10} - 8{,}000$$

TABLE III.2

TABULAR METHOD FOR SPLICE ANALYSIS, ACCOUNTING FOR APPLIED
AXIAL END OR INTERMEDIATE (SHEAR FLOWS) LOADS AND ATTACHMENT "SLOP"

1	2	3	4 *	5	6	7	8	9	10 **	11	12	13	14	15	16	
FASTENER OR STATION	DIFF IN STRAIN	FASTENER "SLOP"	FASTENER STRAIN (SHEAR)	FASTENER SPRING CONST.	FASTENER LOAD	MEMBER D LOAD	MEMBER D SPRING CONST.	MEMBER D STRAIN	INTERM. LOADS	ACCUM. INTERM LOADS	ACCUM. APPLIED LOADS	LOAD IN MEMBER S	MEMBER S SPRING CON.	MEMBER S STRAIN	DIFF. IN STRAIN	
	$\Delta(\delta_S-\delta_D)$	Δc	$\Delta\delta_F$	k_F	P_F	P_D	k_D	δ_D	$q_d \times L_n$			P_S	k_S	δ_S	$\delta_S-\delta_D$	
n	$(2)_{n-1}-(16)_{n-1}$	GIVEN	$(2)\pm(3)$ *	GIVEN	$(4)\times(5)$	$\sum(6)$	GIVEN	$(7)/(8)$	GIVEN	$\sum(10)$	$Q_L+(11)$	$(12)-(7)$	GIVEN	$(13)/(14)$	$(15)-(9)$	
	$\times 10^6$	$\times 10^6$	$\times 10^6$	$\times 10^{-6}$			$\times 10^{-6}$	$\times 10^6$					$\times 10^{-6}$	$\times 10^6$	$\times 10^6$	
1	4000	3000	1000	.80	800	800	4.7	170	0	0	8000	7200	4.70	1532	1362	FIRST TRIAL
2	2638	0	2638	"	2110	2910	"	619	"	"	"	5090	"	1184	565	
3	2073	0	2073	"	1658	4568	"	973	"	"	"	3432	"	729	-244	
4	2317	0	2317	"	1852	6420	"	1365	"	"	"	1580	"	335	-1029	
5	3346	0	3346	"	2680	9100	"	1934	"	"	"	-1100	"	-234	-2168	
6	5514	0	5514	"	4410	13510	"	2870	"	"	"	-5510	"			
7	9554	0	9554	"	7640	21150	"	4500	"	"	"	-13150	"	-2790	-7290	
8	16844	0	16844	"	13460	34610	"	7360	"	"	"	-26610	"	-5660	-13020	
9	29864	0	29864	"	23900	58510	"	12440	"	"	8000	-50510	"	-10740	-23180	
10	53044	0	53044	"	42500	101010	"	--	--	--	--	--	"	--	--	
1	3520	3000	520	.80	416	416	4.7	89	0	0	8000	7534	4.70	1614	1525	FINAL TRIAL
2	1995	0	1995	"	1596	2012	"	428	"	"	"	5988	"	1273	845	
3	1150	0	1150	"	920	2932	"	624	"	"	"	5068	"	1078	454	
4	696	0	596	"	556	3488	"	743	"	"	"	4512	"	962	219	
5	477	0	477	"	381	3869	"	823	"	"	"	4131	"	880	57	
6	420	0	420	"	336	4205	"	895	"	"	"	3795	"	808	-87	
7	507	0	507	"	405	4610	"	980	"	"	"	3390	"	722	-258	
8	765	0	765	"	610	5220	"	1110	"	"	"	2780	"	593	-517	
9	1282	0	1282	"	1025	6245	"	1325	"	"	8000	1755	"	374	-951	
10	2233	0	2231	"	1780	8025	--	--	--	--	--	--	--	--	--	

* (3) Makes (4) less than (2), but cannot reverse the "sign". (i.e., can "reduce" (2) only to zero).
** The intermediate load is due to either an applied shear flow, qa, or a local applied axial load. Assume value for $(6)_{n=1}$ & complete table. For correctly assumed $(6)_{n=1}$, $(7)_{n=N} = (12)_{N-1}$

This relatively simple analysis is all that is necessary for those installations where all internal loads are in the elastic range (i.e., where no yielding is to be allowed, usually at limit load). The same note on p. 37 regarding "large slop" at Fastener #1 applies here also.

Some labor-saving "short-cuts" in determining the internal loads of doubler and splice installations are presented in Appendix I, Article AI.2.

III.4 COMPARISON OF DOUBLERS AND SPLICES

It is helpful to keep in mind that there are two basic differences between doublers and splices

a. They have different purposes

(1) A splice's function is to transfer a given load. It is kept as as short as possible in accomplishing this.

(2) A doubler's function is to pick up load (and relieve another member). In order to do this efficiently it must have some considerable length, although this is kept to a minimum. Therefore doublers are, by nature, relatively long members compared to splices.

b. As can be seen from an inspection of the results of Table III.1 and III.2, Column 6

(1) The fastener loads in splices can be made to approach a somewhat uniform distribution efficiently since they are all acting in one direction (unless unusual intermediate applied loads are present)

(2) In a doubler, however, the fastener loads form two groups acting in opposite directions to load and unload the doubler. Thus, the fastener loads will be larger at the ends and vanish at the center where the relative displacement between members D and S is zero. They will not, efficiently, approach uniformity as in the case of the splice.

These facts are illustrated in Figure III.8

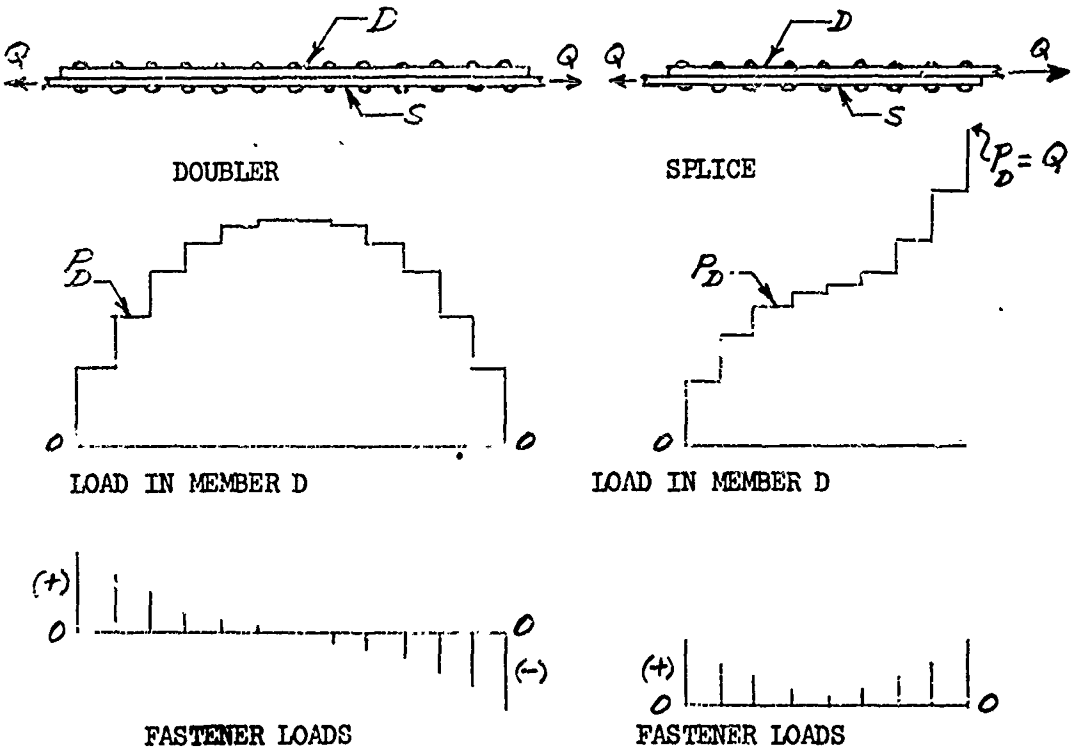
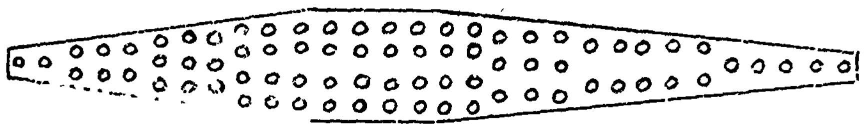


Figure III.8. Comparison of Internal Loads in Typical Doublers and Splices

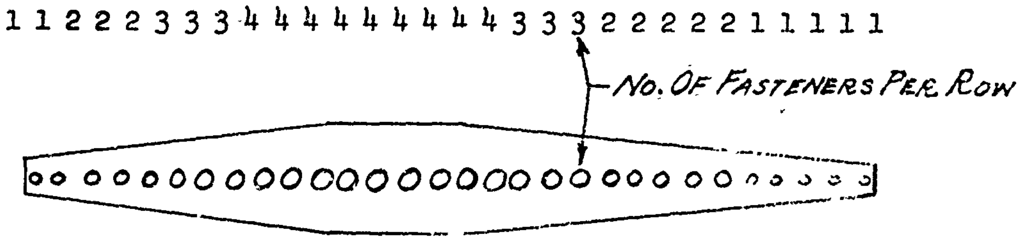
III.5 GROUPING STRUCTURAL ELEMENTS

When there is more than one fastener in a row (normal to the loading, or to the axial direction) the spring constants of the individual fasteners in the row can be simply added together and considered as one fastener. The spring constants of the axial members are calculated in terms of their "adjusted" net average cross-sectional area, and the effect of more than one fastener is considered, as illustrated in Section V, Figure V.4. This substitution is illustrated in Figure III.9.

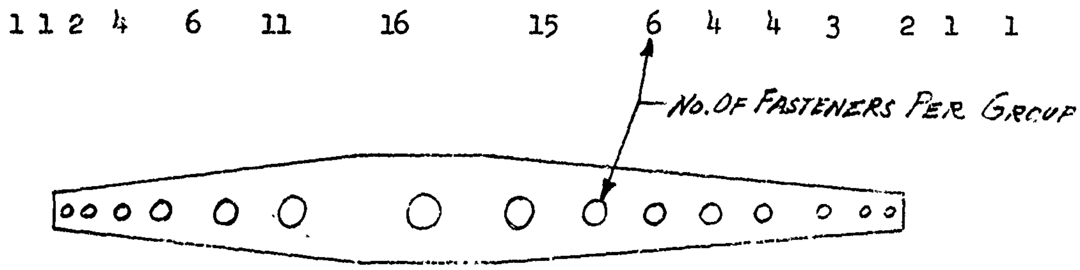
Frequently, however, in the case of doubler installations there are too many rows of fasteners for a hand analysis to include all of them, and it is necessary to group, or "lump", two or more rows together as one row, or one fastener actually. Since the end fasteners are the most highly loaded it is best to do the least grouping at the ends and the most at the middle. Figure III.9 illustrates how this is carried out.



Actual Doubler
(a)



Conversion Of Rows To Single Equivalent Fasteners
(b)



Grouping Of Fasteners
For Analysis Purposes
(c)

Figure III.9 Grouping Of Fasteners To Facilitate Analysis

As seen, the doubler having 30 rows of fasteners (a total of 77 fasteners) would be first considered, for analysis purposes, as having 30 "equivalent" fasteners as in (b). Then, since these are too many for a hand analysis, they would be "lumped" into say, 15 groups, that is, into 15 equivalent fasteners for a hand analysis. In either case, (b) or (c) the equivalent fastener has a value of k_F obtained as the sum of the individual values of k_{F_n} which it replaces ($= \sum k_{F_n}$). It can be seen that the largest grouping in (c) is done in the middle portion, where the fasteners are strained the least. The location of each group (or equivalent fastener) in (c) is the "centroid" of the fasteners in the group, based on their spring constants. The spring constants of the axial members, D and S, are obtained from (c) but include the effect of the fastener holes as they actually exist, in (a). The equivalent structure in (c) is then analyzed using the method as discussed.

Once the fastener group loads are determined they can be distributed to the individual fasteners making up the group on the basis of fastener spring constants, since fasteners having different values of k_F are sometimes grouped together. That is,

$$P_{F_n} = P_{Group}\left(\frac{k_{F_n}}{\sum k_{F_n}}\right)$$

This method of grouping can also be used should there be too many rows for the computer routine to handle, as discussed in Section IV.

III.6 FASTENER LOADS IN THE PLASTIC RANGE

In the previous discussions and examples it has been assumed that the fastener spring constants, k_F, are known as supplied data. However, as discussed in Section V and illustrated in Figure V.2, these values may not be constant. Therefore, if the applied loads are large enough, a procedure is necessary that accounts for the reduction in k_F, at each affected fastener in the "plastic" range. (A review of Section V is helpful at this stage).

This can be done by using the previous tabular method of analysis but carrying out separate analyses for successive increments of the applied load until their total equals the applied load. That is, the method of superposition is used. During each increment of applied load the values of k_F will be assumed to be constant, but they may change for successive increments. The procedure is as follows:

a. The maximum load to which any fastener is allowed to be subjected must be determined. This value will be established by either a fatigue or yielding requirement, or else as the ultimate load for the fastener sheet combination. (This is discussed further in Section VIII).

b. The load-deflection curve (for each type of fastener) is divided into several straight line portions that

approximate it as shown in Figure III.10. Although not necessary, it may be convenient to use equal increments on the P scale, as shown, for all but the first increment.

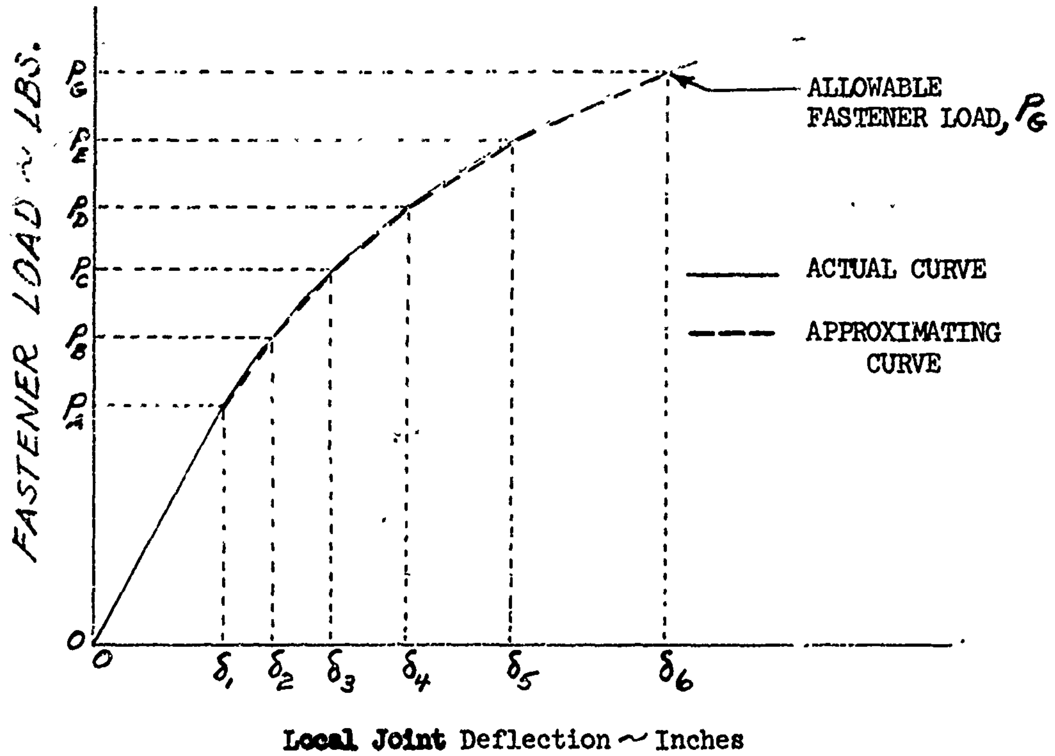


Figure III.10 Division Of A Fastener-Sheet Load-Deflection Curve Into Linear Increments

Six increments are shown in Figure III.10 since this number is used in the computer routine. (A lesser number of increments, only 2, are used for hand analyses as illustrated in the following example problem). The increments are chosen as follows. The first increment, from 0 to P_A, includes the linear portion. The difference in load between P_A and the maximum value to be allowed, P_G, is divided into 5 equal load increments and the corresponding deflections, δ_n, are determined. Then the value of k_F for each linear portion is calculated as

$$k_{F_{A,\,B,\,C\,--}} = \left(\frac{\Delta P}{\Delta \delta}\right)_{A,\,B,\,C\,-\,-\,-}$$

c. Assuming all fastener spring constants to have their initial (elastic) values, k_{F_A}, the loads in the fasteners for the full applied load, Q_L, are determined by the conventional tabular analysis.

d. The largest resulting load, $P_{F_{n_1}}$, at each different type of fastener-sheet combination is examined in light of its load deflection curve (Figure III.10). If any of the fasteners are loaded above their P_A values, all of the results in c. above, including the value Q_L are reduced by the fraction, $P_A/P_{F_{n1}}$. $P_A/P_{F_{n1}}$ is the smallest fraction obtainable from the results. The first applied load increment, ΔQ_1, is then calculated as

$$\Delta Q_1 = Q_L \left(\frac{P_A}{P_{F_{n1}}} \right)$$

e. Steps c and d are repeated for an applied load of $Q_L - \Delta Q_1$ and a new set of loads, $P_{F_{n2}}$, is obtained; but this time k_{F_A} is used for all fasteners except that one in d above that has reached its limit of P_A. For this fastener k_{F_B} is used in the analyses. The sum of the loads at each fastener is then computed. Examining the results as before, another fraction, $\frac{P_A - P_{F_n}}{P_{F_{n2}}}$, is obtained. However, it is possible that the same fastener may again reach a new limit, P_B, and that the fraction $\frac{P_B - P_{F_n}}{P_{F_{n2}}}$ here may be the smallest. The corresponding loading increment is calculated as

$$\Delta Q_2 = (Q_L - \Delta Q_1) \left(\frac{P_A - P_{F_{n1}}}{P_{F_{n2}}} \right),$$

$$\text{Or as} \quad \Delta Q_2 = (Q_L - \Delta Q_1) \left(\frac{P_B - P_{F1_1}}{P_{F1_2}} \right)$$

f. Steps c and d are repeated again, repetitively, until after m sets of calculations the sum of the increments of ΔQ_m, or $\sum \Delta Q_m$, is equal to the applied load, Q_L. The fastener load distribution will be the sums of those obtained in each increment, that is, those obtained in each analysis after ratioing down the results. The same applies to the axial loads in the members D and S.

g. If any fastener reaches its maximum allowable load before $\sum \Delta Q_m = Q_L$ then $\sum \Delta Q_m$ is the max. load the structure can take. Summarizing, for any analysis increment, m, the following steps will be used.

(1) Calculate $Q_m = Q_L - \sum^{m-1} \Delta Q_m$, and if an applied shear flow, q_a, is present

$$q_m = q_a \times \frac{Q_m}{Q_L}$$

(2) Calculate the internal load distribution by a conventional tabular analysis, for the applied loads Q_m and q_m (if present).

(3) Determine the smallest ratio

$$r_{n_m} = \frac{P_N - \sum^{m-1} P_{F_{n_m}}}{P_{F_{n_m}}}$$

where N refers to the selected P_N values as in Figure III.10. If all values of r_{n_m} are greater than 1.0, then $r_{n_m} = 1.0$ is used.

(4) Calculate the increment of applied load for this analysis, m, as

$$\Delta Q_m = Q_m \times r_{n_m}$$

and

$$\Delta q_m = q_m \times r_{n_m}$$

(5) Calculate the increments of fastener loads for this analysis, m, as (for each fastener)

$$\Delta P_{F_{n_m}} = P_{F_{n_m}} \times r_{n_m}$$

(6) Calculate the increments of load in the members D and S as

$$\Delta P_{S_{n_m}} = P_{S_{n_m}} \times r_{n_m}$$

$$\Delta P_{D_{n_m}} = P_{D_{n_m}} \times r_{n_m}$$

Steps (1) through (6) can then be repeated in the next analysis, m + 1, etc, until $\sum^{m} \Delta Q_m = Q_L$.

The analysis can be carried out most easily by using a tabular form for the calculations. The details of this are illustrated in the following example problem.

For cases where slop is present an additional refinement is necessary as discussed at the end of this article.

EXAMPLE PROBLEM

A doubler is attached to a base structure as shown in Figure III.11a. The fastener load-deflection curve is shown in Figure III.11b. Determine by hand analysis:

a. the internal load distribution corresponding to the applied load of 44,800#

b. the maximum value the applied load could have if the allowable fastener load is 6450#, as shown in Figure III.11b, and the corresponding internal loads.

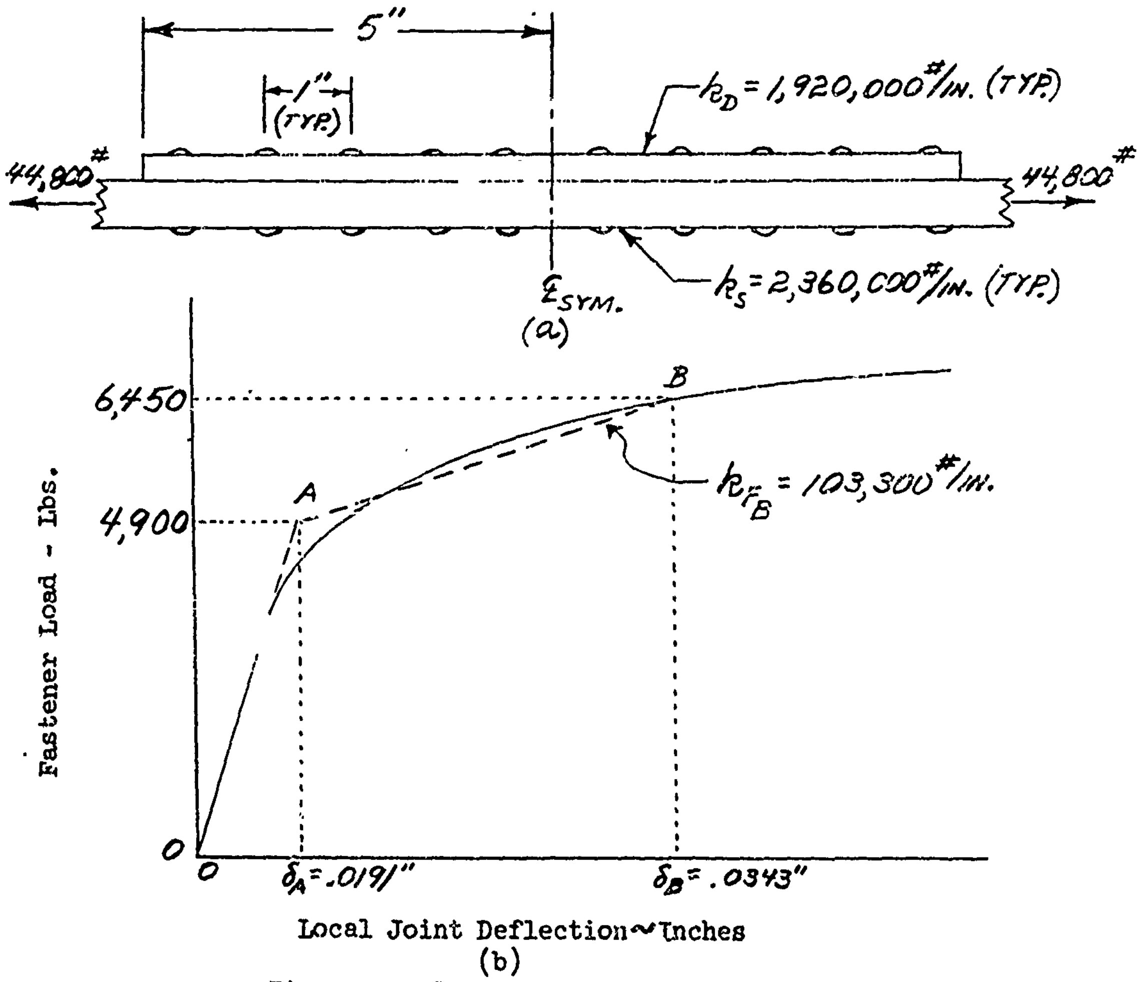


Figure III.11 A Doubler Installation

a. The analysis is carried out in Table III.3 as follows:

(1) The actual load-deflection curve of Figure III.11b is replaced by one consisting of 2 straight lines, as shown by the dashed lines. This has been done in such a manner as to obtain approximately the same area under each curve. The maximum (allowable) value of P_F is 6450# as arbitrarily specified above. Hence, it is seen that for all fasteners P_A = 4,900# and P_B = 6,450#. The two resulting spring constants for the fasteners are found to be (the "slopes")

$$k_{F_A} = 256{,}000 \ \#/\text{in} \quad \text{and} \quad k_{F_B} = 103{,}300\#/\text{in}$$

(2) A conventional tabular hand analysis is then carried out to determine the internal load distribution in the structure for the applied load of 44,800# and for k_1---k_5 = 256,000 #/in. This is referred to as the "first unit solution" and the results are entered in Col (2). Only the doubler and base structure internal loads in the center elements, P_{D_5} and P_{S_5} are shown, to save space.

(3) The limiting load levels for the fasteners for this first analysis are shown in Col. (3) as 4900# (which is P_A). The limiting value of Q_L is the applied value of 44,800#.

(4) The possible limiting ratios are calculated in Col. (4) .

(5) The smallest value in Col. (4) (r_1 = .646) is then applied to the internal loads of Col. (2) to obtain the actual loads making up the first so-called "increment" of loading. This increment is based upon k_1 --- k_5 = 256,000#/In. The results are listed in Col. (5) . Col. (6) is the sum of all previous increments, which is identical to the first increment of Col. (5) . This brings the first fastener up to its max. value of load, P_{F_1} = 4900, that is consistent with k_{F_1} = 256,000#/in. This is seen to correspond to an applied load increment of 28,900#.

(6) A second conventional tabular hand analysis is then made for the remaining applied load of 44,800 - 28,900 = 15,900# and for k_{F_1} = 103,300 and k_{F_2} --- k_{F_5} = 256,000#/in. This is called the "second unit solution" and the results are entered in Col. (7).

TABLE III.3

DETERMINATION OF INTERNAL LOADS IN THE PLASTIC RANGE

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
LOAD	FIRST UNIT SOLUTION	LIMITING LOAD LEVEL	POSSIBLE LIMITING RATIOS	FIRST LOADING INCREM.	SUM OF LOADS	SECOND UNIT SOLUTION	LIMITING LOAD LEVELS	POSSIBLE LIMITING RATIOS	SECOND LOADING INCREM.	SUM OF LOADS	THIRD UNIT SOLUTION	LIMITING LOAD LEVELS	POSSIBLE LIMITING RATIOS	THIRD LOADING INCREM.	SUM OF LOADS	FOURTH UNIT SOLUTION
			r_1	Δ_1	$\Sigma\Delta$			r_2	Δ_2	$\Sigma\Delta_2$			r_3	Δ_3	$\Sigma\Delta$	
	$Q_1 = Q_L$ FROM TABULAR ANALYSIS	$Q = Q_{(2)}$ P_N FROM FIG. III.11b	$\frac{(3)}{(2)}$	$(4)_{MIN.} \times (2)$	(5)	$Q_2 = Q_{(2)} - Q_{(6)}$ FROM TABULAR ANALYSIS	$Q_{Lim} = Q_{(2)}$ P_N FROM FIG. III.11b	$\frac{(8) - (6)}{(7)}$	$(9)_{MIN.} \times (7)$	(6) + (10)	$Q_3 = Q_{(2)} - Q_{(11)}$ FROM TABULAR ANALYSIS	$Q_{Lim} = Q_{(2)}$ P_N FROM FIG. III.11b	$\frac{(13) - (11)}{(12)}$	$(14)_{MIN.} \times (12)$	(11) + (15)	$Q_4 = Q_{(2)} - Q_{(16)}$
	$k_1 - - k_5 = 256{,}000$					$k_1 = 103{,}300$, $k_2 - - - k_5 = 256{,}00$					k_1 & k_2 = 103,300, $k_3 - - k_5 = 256{,}000$					
Q_L	44,800	44,800	1.000	28,900	28,900	15,900	44,800	1.000	14,830	43,730	1070	44,800	1.000	1070	44,800	0
P_{F1}	7,592	4,900	.646	4,900	4,900	1,404	6,450	1.103	1,310	6,210	107	6,450	2.25	107	6,317	0
P_{F2}	4,568	4,900	1.074	2,950	2,950	2,090	4,900	.933	1,950	4,900	70	6,450	22.1	70	4,970	0
P_{F3}	2,649	4,900	1.848	1,710	1,710	1,212	4,900	2.63	1,130	2,840	101	4,900	20.4	101	2,941	0
P_{F4}	1,371	4,900	3.57	885	885	627	4,900	6.40	585	1,470	52	4,900	55.9	52	1,522	0
P_{F5}	423	4,900	11.57	273	273	194	4,900	23.8	181	454	16	4,900	278.0	16	470	0
P_{D5}	16,603	--	--	10,718	10,718	5,527	--	--	5,156	15,874	346	--	--	346	16,220	0
P_{S5}	28,197	--	--	18,182	18,182	10,373	--	--	9,674	27,856	724	--	--	724	28,580	0

(7) The remaining columns are then completed in a similar manner to that for Col. 2 - Col. 6. It is seen in Col. 14 that the limiting ratios for the fasteners are all greater than 1.0 Hence, the value $r_3 = 1.0$ is used and Col. 15 is identical to Col. 12. The final loads are then those obtained in Col. 16 since Col. 17 shows Q_X to be zero.

Although this analysis happened to be completed in only three increments, other configurations might require more. Such could have happened in this case if the fasteners were more closely spaced or if the fasteners were less stiff initially than shown.

b. The maximum "allowable" applied load, Q_L, and the corresponding internal loads can be calculated by revising Col. 14 - 18 as shown in Table III.4.

(1) Since the load Q_L is to be determined no limiting ratio is specified for it in Col. 14.

(2) The smallest of the remaining limiting ratios in Col. 14, or 2.25, is then applied to the values of Col. 12 to obtain the values of Col. 16. Col. 16 then gives the allowable applied load Q_L (=46,140#) and the corresponding internal loads. It is seen that, in this case, it is the end fastener that reaches its allowable load of 6450# first and limits the load carrying ability of the structure.

TABLE III.4

DETERMINATION OF THE ALLOWABLE APPLIED LOAD FOR THE STRUCTURE

1	2 - 11	12	13	14	15	16
Load		THIRD UNIT SOLUTION	LIMITING LOAD LEVELS	POSSIBLE LIMITING RATIOS,	THIRD LOADING INCR'M'T,	SUM OF LOADING INCR'M'T,
	Same As Table III.3	$Q_3 = Q_{(2)} - Q_{(11)}$ k_1 & k_2 = 103,000 k_3---k_5= 256,000	P_N FROM Fig. III.11b			
		FROM TABULAR ANALYSIS	$Q_{(2)}$	$\frac{(13) - (11)}{(12)}$	(14) + (12) MIN.	(11) + (15)
Q_L		1070	--	--	2,410	46,140
P_{F1}		107	6,450	2.25	240	6,450
P_{F2}	Same	70	6,450	22.1	158	5,058
P_{F3}	As	101	4,900	20.4	227	3,067
P_{F4}	Table III.3	52	4,900	65.9	117	1,587
P_{F5}		16	4,900	278.0	36	490
P_{D5}		346	--	--	778	16,652
P_{S5}		724	--	--	1631	29,487

The problem of Table III.4 was repeated (by computer) using a fastener load-deflection curve consisting of 6 straight lines. The results are compared with the previous ones in Table III.5. It is seen that, in this particular case, the difference in results is negligible from an engineering standpoint. This is believed to be true in general for fasteners having a significant initially linear portion on the load-deflection curve.

TABLE III.5

COMPARISON OF RESULTS FROM HAND AND COMPUTER ANALYSES

LOAD	RESULTS USING 2 STRAIGHT LINE CURVE (TABLE III.4)	RESULTS USING 6 STRAIGHT LINE CURVE (BY COMPUTER)
Q_L	46,140	45,986
P_{F1}	6,450	6,450
P_{F2}	5,058	4,949
P_{F3}	3,067	3,080
P_{F4}	1,587	1,593
P_{F5}	490	492
P_{D5}	16,652	16,564
P_{S5}	29,488	29,422

Although not illustrated, the same general procedure can be used for the case of a splice having fastener loads in the plastic range. That is, the same steps as outlined for the doubler would be taken. The only difference would be that the unit solutions of Table III.3 would be made for a splice.

This article has considered only the case of the fasteners "going plastic". Although less likely, the doubler or the base structure elements might also be loaded into the plastic range. In such cases the same general procedure would apply, but the stress-strain curve of the sheet material would be used (similar to the fastener load-deflection curve) and "replaced" by straight line segments. That is, the tangent modulus, E_t, would be used to calculate k_D or k_S in the non-linear portion. Any such doubler or base structure elements would, for example, be included in Col. 1 of Table III.3 and they, also, would have values for Col. 2, 3 and all subsequent columns, just as did the fasteners in the example illustrated.

The method of this article has not included provision for slop. If slop is present a slight additional refinement must be made. This is discussed and illustrated in Appendix I, Article AI.3.

III.7 SUCCESSIVE LOADINGS IN THE PLASTIC RANGE

When the applied loading results in any fastener(s) being loaded in the plastic range, permanent set will occur. Therefore, when the applied load is removed there will remain some distribution of

internal, or residual, loads in the structure. That is, the structure will be "pre-loaded". Any successive applied load will start from this basis. Thus, it may be necessary to be able to predict these residual loads in order to obtain the true internal load distributions corresponding to subsequent applied loads. This might be necessary in a fatigue life evaluation, particularly. A method of accomplishing this follows. *

Assuming that a doubler installation has been loaded so that one or more fasteners is in the plastic range, when the applied load is removed these fasteners will unload at an essentially constant rate (lbs/in). This rate will be very nearly the same as the slope of the initial linear portion of the load-deflection curve, as evident from experiments. This is illustrated in Figure III.12 and is analagous to what occurs when any ductile material is loaded beyond the proportional limit. (Actually the line B-δ_1 or C-δ_2 is a hysteresis "loop" and B-δ_1 and C-δ_2 have a significantly steeper slope than does OA. But this is ignored in the suggested analysis and is discussed in Sections V and VII)

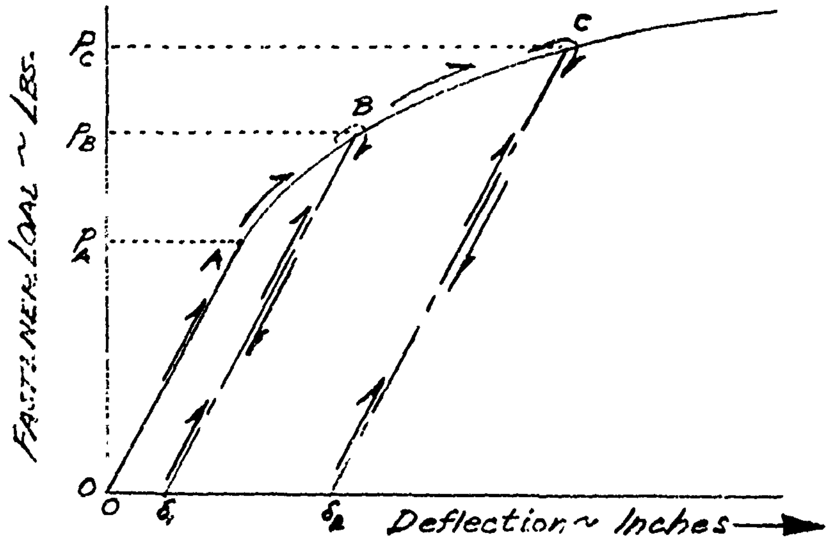


Figure III.12 Loading and Unloading in the Plastic Range

That is, if a fastener were initially loaded beyond the elastic (linear) range, P_A, to say, P_B, it would return to a residual strain, δ_1, when unloaded. Then if loaded again to a higher load level, P_C, it would, essentially, follow the line δ_1-B-C and upon being unloaded it would follow the line C-δ_2 to a permanent set of δ_2 when $P = 0$. The lines δ_1-B and δ_2-C are essentially parallel to the initial linear portion, O-A. The main point is that in unloading the fastener load decreases at a rate (lbs per inch of deflection) that corresponds, essentially, to the initial (elastic) slope of its load-deflection curve and follows this slope in loading up again.

* As discussed in Sections V and VII some permanent set will always occur, even at low load levels in the so-called elastic range, due to the "seating" of the fastener in the holes.

The residual internal loads can therefore be calculated by a superposition precedure as follows:

a. Calculate the set of internal loads, using the specified applied load but assuming that the spring constants, k_{F_n}, for all fasteners are the initial (elastic) values.

b. Subtract these values from those obtained in the plastic analysis (as in Article III.6). The resulting values are the residual loads in all members.

Table III.6 illustrates the determination of the residual loads for the doubler of Art. III.6, Figure III.11 loaded into the plastic range.

TABLE III.6

DETERMINATION OF RESIDUAL LOADS

①	②	③	④
LOAD	RESULTS OF THE PLASTIC ANALYSIS	ELASTIC ANALYSIS FOR Q_L = 44,800 k_1---k_5= 256,000 #/in	RESIDUAL LOADS
	TABLE III.3, COL. ⑱	TABLE III.3, COL. ②	② - ③
Q_L	44,800	44,800	0
P_{F1}	6,317	7,592	-1,275
P_{F2}	4,970	4,568	402
P_{F3}	2,941	2,649	292
P_{F4}	1,522	1,371	151
P_{F5}	470	423	47
P_{D5}	16,220	16,603	-383
P_{S5}	28,580	28,197	383

Then for any subsequent applied loading that does not exceed the original applied load the internal loads are obtained by

a. Calculating the load distribution assuming that the spring constants for all fasteners are the initial (elastic) values.

b. Adding the residual loads to the values obtained above, to obtain the true internal load distribution.

If a subsequent applied load is greater than all previous ones, then a "new" plastic analysis is simply carried out as discussed in Article III.6. The residual loads due to this will then be the basis for all lesser subsequent applied loads.

Table III.7 illustrates the determination of the true internal load distribution for subsequent loadings. The case illustrated is for an applied load, Q_L = 22,400 #, a previous load having been the 44,800 # value in Table III.6.

TABLE III.7

DETERMINATION OF SUCCESSIVE LOADS IN THE PLASTIC RANGE

LOAD	ELASTIC ANALYSIS FOR Q_L = 22,400 # k_1---k_5= 256,000 #/IN $\frac{22,400}{44,800}$ x COL. 2, TABLE III.3	RESIDUAL LOADS TABLE III.6, COL. 4	TRUE INTERNAL LOAD DISTRIBUTION 2 + 3
Q_L	22,400	0	22,400
P_{F1}	3,796	-1,275	2,521
P_{F2}	2,284	402	2,686
P_{F3}	1,324	292	1,616
P_{F4}	685	151	836
P_{F5}	211	47	258
P_{D5}	8,302	-383	7,919
P_{S5}	14,099	383	14,482

Additional subsequent applied loads up through 44,800#, would be dealt with similarly.

The above illustration was for a doubler configuration. The same procedure would be used for a splice, however.

The method of this article has not included provisions for including slop. If slop is present a slight additional refinement must be made. This is discussed and illustrated in Appendix I, Article AI.3.

III.8 MULTIPLE DOUBLER AND SPLICES

As specified earlier, the specific methods of this report apply only to doublers or splices consisting of a single lap or a single sandwich configuration. Occasionally, however, the situation may arise where there are several axial members. This would represent a case of multiple or "stacked" members as illustrated in Figure III.13.

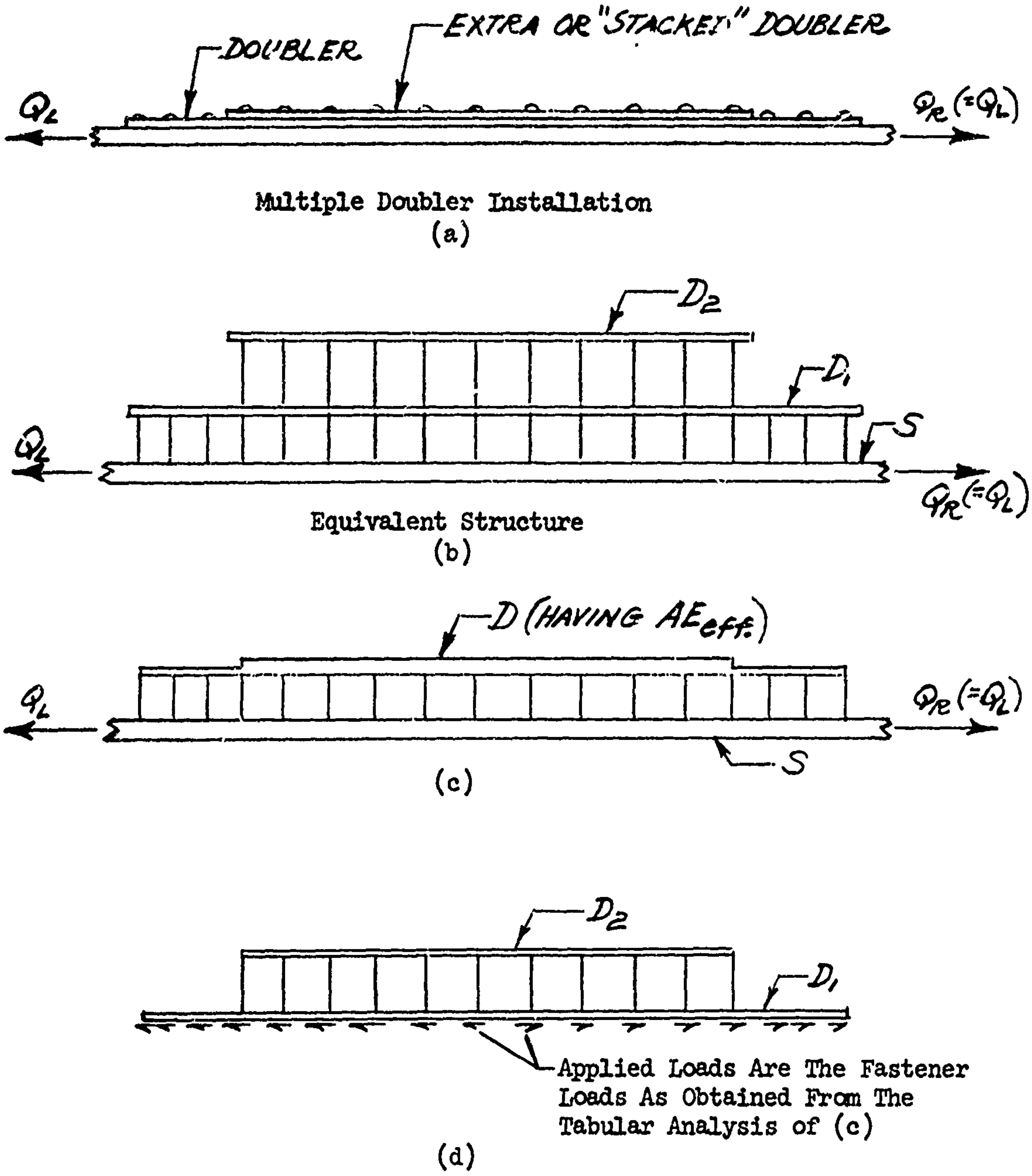


Figure III.13 A Multiple Doubler Installation

The actual structure is shown in (a) and the equivalent structure for purposes of analysis in (b). The distribution of fastener loads and the loads in the members could be determined most directly in such a case by using the analog method discussed in Section 5.0. If this is not available an approximate fastener load distribution can be obtained by successive trials using the basic method of this report as follows:

a. Combine the stacked doublers D_1 and D_2 into one member, D, (by adding the k values) as in Figure III.13c. This assumes the fasteners between them to be rigid.

b. Determine the corresponding fastener loads between this assumed member, D, and the base structure, S, in the conventional tabular manner. Note the strains, Col. 9 of the table.

c. Then consider only the two doublers, as they actually exist, to be a structure subjected to the loads of (b) above, applied to the member D_1, as in Figure III.13d.

d. Determine the internal loads for this configuration and loading and also note the strains in the member D_1 Col. 15 of the table. Member D_1 is the "base structure" in this analysis.

e. Calculate an effective k_D value for the combined members D_1 and D_2 using the member strains from (b) and (d) above as follows:

For any segment the effective k_D of the combined members is taken as

$$(k_D)_{\text{eff.}} = (k_D)_{\text{assumed}} \times \left(\frac{\delta_D}{\delta_{D_1}}\right)$$

f. Repeat steps (b) through (e) using $(k_{D_2})_{\text{eff.}}$ from step (e) above in step (b). Then repeat again as necessary until the strains obtained in (d) are sufficiently identical to those in (b), that is, until at each element, D_n and D_{1_n}

$$\delta_{D_n} = \delta_{D_{1_n}}$$

It can be seen that this involves considerably more effort than for a single doubler, particularly where hand analysis is used. A rougher estimate can, of course, be obtained simply by carrying out steps (a) and (b) only one time. This assumes the doublers to be one integral member and therefore results in the fastener loads and the doubler load being larger than they actually are.

Only the case of one "extra" doubler has been illustrated. The same approach could be used if more than one were present. However, the labor would increase significantly since the steps outlined would have to be made for each "pair" of doublers, successively, and more than two sets of fastener loads would have to be sufficiently identical in the successive analyses.

EXAMPLE PROBLEM.

Determine the internal loads in the structure shown in Figure III.14a, where 2 doublers (a "stacked" arrangement) are attached to a base structure.

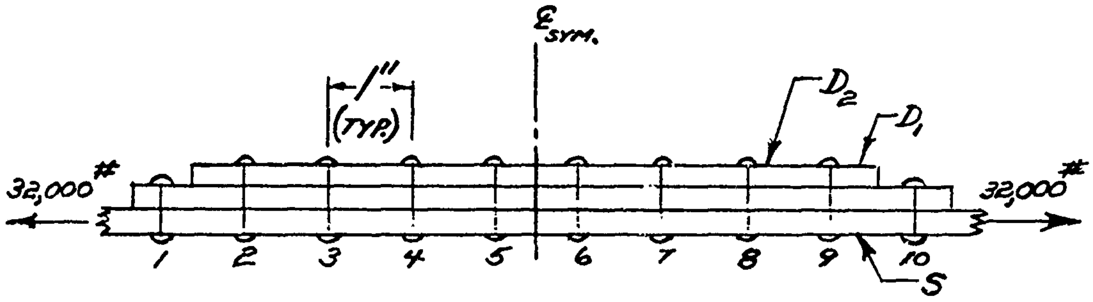


Figure III.14a. A Multiple Doubler Installation

The spring constants of the parts are

(a) $k_{F_n} = .47 \times 10^6$ for all fasteners, and

(b) $k_{S_n} = 2.47 \times 10^6$, $k_{D1_n} = 2.47 \times 10^6$, $k_{D2_n} = 1.23 \times 10^6$

Proceeding according to the previously outlined steps:

a. The two doublers, D_1 and D_2, are considered to be one integral member, D, as in Figure III.13c, having

$$k_{D_n} = k_{D1_n} + k_{D2_n}$$

b. A tabular analysis is then made (as in Article III.2) to determine the internal loads in this structure, D and S, and also the strains in the member D. The results of this analysis are shown in Table III.8 including the resulting strains in member D. Since the structure is symmetrical only half of it is presented.

TABLE III.8

RESULTS OF STEPS a AND b, FIRST TRIAL

ELEM.	P_F	P_D	k_D	δ_D
(RESULTS OBTAINED FROM A TABLE SIMILAR TO III.1)				
1	7816	7816	2.47×10^6	.00317
2	4700	12516	3.70×10^6	.00338
3	2590	15106	"	.00408
4	1290	16396	"	.00443
5	399	16795	"	.00454

c. The two doublers and their attachments are then considered to be a structure subjected to the set of applied loads, P_F, as shown in Figure III.14b.

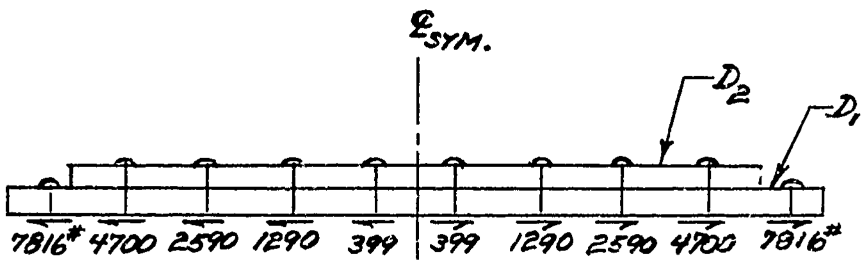


Figure III.14b Loading Applied to the Multiple Doublers

d. An analysis of this structure and loading (as in Article III.2) gives the results shown below, including the strains in the member D_1. Note that only elements 2 through 9, common to D_1 and D_2, are involved in this analysis, as indicated in Table III.9.

TABLE III.9

RESULTS OF STEPS c AND d, FIRST TRIAL

ELEM.	P_{F_1}	P_{D_1}	k_{D_1}	δ_{D_1}
(RESULTS OBTAINED FROM A TABLE SIMILAR TO III.1)				
1	--	7816	2.47×10^6	.00317
2	2409	10107	"	.00409
3	1410	11287	"	.00461
4	719	11858	"	.00408
5	198	12059	"	.00488

Note that the values δ_{D1_n} differ considerably from δ_{D_n} (previous).

e. An effective k_D is then calculated for each of the combined doubler elements as

$$k_{D_{eff.n}} = k_{D_n} \times \frac{\delta_{D_n}}{\delta_{D1_n}}$$

where k_{D_n} is the value in the previous step a. This is shown in Table III.10.

TABLE III.10

RESULTS OF STEP e, FIRST TRIAL

ELEM.	k_D	δ_D	δ_{D_1}	$k_{D_{eff}} = k_D \times \frac{\delta_D}{\delta_{D_1}}$
1	2.47×10^6	.00317	.00317	2.47
2	3.70×10^6	.00338	.00409	3.06
3	"	.00408	.00461	3.28
4	"	.00443	.00480	3.41
5	"	.00454	.00488	3.44

steps (b) through (e) are then repeated using the values of $k_{Deff.}$ in step (b). The results are summarized below.

TABLE III.11

RESULTS OF STEPS b THROUGH d, SECOND TRIAL

ELEM.	STEP b RESULTS				STEP c & d RESULTS			
	P_F	P_D	$k_{Deff} \times 10^{-6}$	δ_D	P_{F1}	P_{D1}	$k_{D1} \times 10^{-6}$	δ_{D1}
1	7634	7634	2.47	.00309	--	7634	2.47	.00309
2	4450	12084	3.06	.00395	2330	9754	"	.00395
3	2510	14594	3.28	.00445	1364	10900	"	.00442
4	1292	15886	3.41	.00466	695	11497	"	.00466
5	418	16304	3.44	.00474	183	11732	"	.00475

Since the strains δ_{D_n} and δ_{D1_n} are essentially identical, it is not necessary to carry out step e and repeat steps b - d again.

The final loads (from steps b - d above) are then as shown in Figure III.14c.

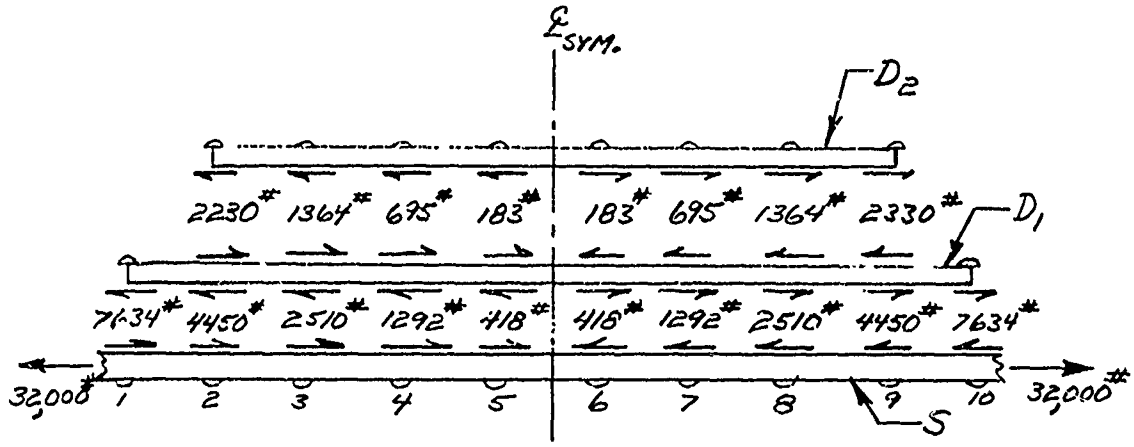


Figure III.14c. Fastener Loads in a Multiple Doubler Installation

Although this analysis was accomplished in only two sets of steps, others might require more than two. A computer program is also presented for this procedure in Section IV and checks the above results quite closely. This routine is, however, limited to only one extra doubler (and does not account for slop or plasticity).

III.9 ANALYSIS FOR THE CASE OF A WIDE BASE STRUCTURE

The previous method of analysis requires only a single definition of A_sE_s for each element of the base structure (and of A_DE_D the doubler elements). From these the spring constants, k_s, are calculated, and used to compute the strain in the members. However, as seen in Equations (31) and (29), it is assumed that only one value of k_s (at each element) applies to all loads acting on the element being considered, as accumulated in Equation (29). This would actually be the case only for relatively narrow base structures (or doublers) having a width of, say, up to 10 times the fastener diameter. When the member is "wide" the fastener loads are not "immediately" effective over the entire cross-section. That is, each fastener load "diffuses" into the base structure (lengthwise) in a manner similar to that considered in evaluating "shear-lag" effects. Therefore, at any element of the base structures, the effective width (and area) is, generally, a different value for each of the fastener loads being accumulated at it in Equation (29). Hence, Equation (31) would be more accurately written as

$$\delta s_n = \frac{QL}{\left(\frac{A_sE_s}{L}\right)_n} + \sum^{n} \frac{q_a\left(\frac{L_{n-1} + L_n}{2}\right)}{\left(\frac{A_sE_s}{L}\right)_n} - \sum^{n} \frac{P_{F_n}}{\left(\frac{A_sE_s}{L}\right)_n} \quad \text{-----(31a)}$$

It is probably sufficiently accurate to deal with the values $(A_sE_s)_n$ in the first 2 terms as discussed in Section V. * But the value of $\frac{A_sE_s}{L}$ in the last term is more accurately evaluated by considering the diffusion mentioned above. This is illustrated in Figure III.15.

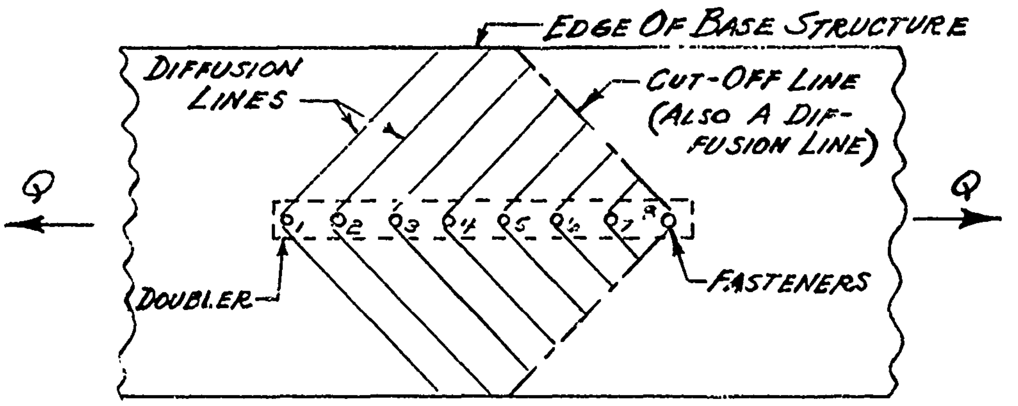


Figure III.15 Doubler Installed on a Wide Base Structure

* There is also a diffusion of any intermediate loads (q_aL_n) into the base structure. However, this effect is not as severe and such loads are not generally present, so the suggested analysis is not further complicated by including it.

The diffusion lines assumed for each of the fastener loads are shown (a 45° slope is arbitrarily used). A "cut-off" line eminating from the last fastener (#8) is shown. This is simply a "reversed" diffusion line at the last fastener. The effective width of the base structure at any element (center) for any fastener load (the last term in Equation 31a) will then be the smallest of the widths between

a. the diffusion lines, or

b. the actual edges of the base structure, or

c. the cut-off lines

Therefore, for each base structure segment there will be a specific width for each fastener load to the left of it. A proper definition of the diffusion lines must be determined experimentally.

The result of this additional refinement (i.e., the various effective widths as defined by the diffusion lines) is to predict smaller fastener loads (and a smaller doubler load) than would otherwise be predicted. However, it does involve considerable additional computation effort, there being essentially 2 extra columns in the table of calculations for each fastener. The following example illustrates the details of the analysis and shows how the basic table of calculations is revised to account for the diffusion effect.

In general it should not be necessary to account for this diffusion effect in the doubler, only in the base structure. This is because the form of the doubler is (efficiently) such as to allow the fastener load to be, essentially, constant over the cross-section. That is, as the doubler widens more fasteners will usually be added, and, more importantly, where the fastener loads are large (at the ends) the doubler is, by nature, narrow rather than wide like the base structure. Similarly, in splices it should not usually be necessary to consider the diffusion effect because of the natural (narrow) form of the members. More specific suggestions for establishing the diffusion lines in practical problems are presented in Appendix I.

EXAMPLE:

A doubler is installed on a wide base structure as shown in Figure III.16.

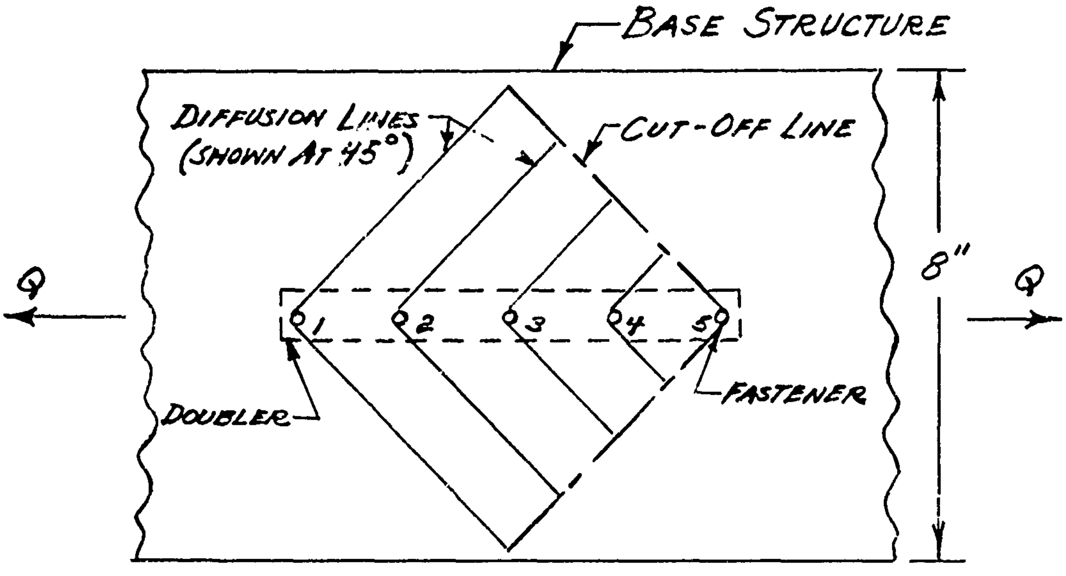


Figure III.16 A Doubler Installed on a Wide Base Structure

The following properties and load are assumed for the example:

$$k_f = 100{,}000\#/\text{in.},\ k_D = \frac{A_D E_D}{L} = 1 \times 10^6,\ k_S = \frac{A_S E_S}{L} = 4 \times 10^6$$

$$A_D E_D = 1 \times 10^6,\ A_S E_S = 4 \times 10^6,\ Q = 40{,}000\ \#$$

For the diffusion lines as assumed in Figure III.16, the effective AE/L of any base structure segment , for each fastener load, n, to its left is shown in Table III.12. These are obtained as previously discussed.

TABLE III.12

BASE STRUCTURE $\left(\frac{AE}{L}\right)_{eff}$ FOR FASTENER LOADS IMPOSED

ELEM.	EFF. $\frac{A_S E_S}{L}$ FOR FASTENER LOADS P_{F_n}			
	P_{F_1}	P_{F_2}	P_{F_3}	P_{F_4}
1	500,000	--	--	--
2	1,500,000	500,000	--	--
3	1,500,000	1,500,000	500,000	---
4	500,000	500,000	500,000	500,000

The analysis is carried out in Table III.13. This table is similar to the conventional one (Table III.1) through Col. 12 . Beginning with Col. 15 , however, additional columns are provided to define the spring constants (AE/L) for the effective widths of the base structure as defined by the diffusion lines. There is a column for each fastener (except the last), Col. 15 through 18 . Then an additional set of columns, 19 through 22 , is provided for the values of strain, P/k. These strains are summed up in Col. 23 and subtracted from the strain (Q/k_{s_0}) in Col. 24 to give the net strain in the base structure at the fastener. The difference in strain between the doubler and the base structure at each fastener is computed in Col. 25 .

The fastener loads are shown in Col. 6 . The final loads should in this example (from symmetry) be symmetrical about the center fastener, #3, and the center fastener load should be zero. This is not quite the case, but is probably due to the assumptions made in accounting for the diffusion effect. However, the method is believed to be suitable for common engineering purposes and is more accurate than ignoring the effects of diffusion altogether. The results obtained when the diffusion effect is ignored are shown in Table III.14, Col. 6 . It is seen that considerably larger fastener loads are predicted in Table III.14.

Some suggested practices for practical design purposes are presented in Appendix I, Articles AI.6 and AI.7. These are based upon the results of the test program and related calculations for doublers on wide base structures.

TABLE III.13
INTERNAL LOAD DISTRIBUTION FOR THE DIFFUSION LINES ASSUMED IN FIGURE III.16

1	2	3	4	5	6	7	8	9	10	11	12	13
ELEM.	Δ DIFF. IN STRAIN $\Delta(\delta_s-\delta_D)$	FASTENER SLOP Δc	FASTENER STRAIN $\Delta\delta_F$	FAST. SPR. CONST. k_F	FAST. LOAD P_F	DOUBLER LOAD P_D	DOUBLER SPRING CONST. k_D	DOUBLER STRAIN δ_D	INT. LOADS	ACCUM. INTERM. LOADS	ACCUM. APP. LOADS	BASE STR. SPR. CONST.
n	(2)$_{n-1}$ − (16)$_{n-1}$	DATA	(2) − (3)	DATA	(4) x (5)	$\sum$ (6)	DATA	(7)/(8)	$q_a x L_n$ DATA	$\sum$ (10)	q_L + (11)	DATA
	x10⁶	x10⁶	x10⁶	x10⁻⁶			x10⁻⁶	x10⁶				x10⁻⁶
FIRST TRIAL												
1	13,100	0	13,100	.100	1,310	1,310	1.000	1,310	0	0	40,000	4.00
2	7,030	0	7,030	.100	703	2,013	1.000	2,013	0	0	40,000	4.00
3	1,420	0	1,420	.100	142	2,155	1.000	2,155	0	0	40,000	4.00
4	-4,800	0	-4,800	.100	-480	1,675	1.000	1,640	0	0	40,000	4.00
5	-9810	0	-9810	.100	-980	695	--	--	0	0	40,000	4.00
SECOND TRIAL												
1	12,900	0	12,900	.100	1,290	1,290	1.000	1,290	0	0	40,000	4.00
2	6,770	0	6,770	.100	677	1,967	1.000	1,967	0	0	40,000	4.00
3	960	0	960	.100	96	2,063	1.000	2,063	0	0	40,000	4.00
4	-5,260	0	-5,260	.100	-526	1,537	1.000	1,537	0	0	40,000	4.00
5	-10,640	0	-10,640	.100	-1064	473	--	--	0	0	40,000	4.00
THIRD TRIAL												
1	12,600	0	12,600	.100	1,260	1,260	1.000	1,260	0	0	40,000	4.00
2	6,380	0	6,380	.100	638	1,898	1.000	1,898	0	0	40,000	4.00
3	400	0	400	.100	40	1,938	1.000	1,938	0	0	40,000	4.00
4	-6,310	0	-6,310	.100	-631	1,307	1.000	1,307	0	0	40,000	4.00
5	-12,380	0	-12,380	.100	-1238	69	--	--	0	0	40,000	4.00
FOURTH (FINAL) TRIAL												
1	12,560	0	12,560	.100	1,256	1,256	1.000	1,256	0	0	40,000	4.00
2	6,326	0	6,326	.100	633	1,889	1.000	1,889	0	0	40,000	4.00
3	317	0	317	.100	32	1,921	1.000	1,921	0	0	40,000	4.00
4	-6,419	0	-6,419	.100	-642	1,279	1.000	1,279	0	0	40,000	4.00
5	-12,585	0	-12,585	.100	-1258	21	--	--	0	0	40,000	4.00

1	14	15	16	17	18	19	20	21	22	23	24	25
ELEM.	BASE STR. PRIMARY STRAIN δ_{s_o}	k_{s_1}	k_{s_2}	k_{s_3}	k_{s_4}	P_1/k_{s_1}	P_2/k_{s_2}	P_3/k_{s_3}	P_4/k_{s_4}	SUM OF LOCAL STRAINS $\sum P_n/k_{s_n}$	NET BASE STR. STRAIN $\delta_{s_{net}}$	DIFF. IN STRAIN $\delta_{s_{net}}-\delta_D$
n	(12)/(13)	DATA	DATA	DATA	DATA	(6)$_1$/(15)	(6)$_2$/(16)	(6)$_3$/(17)	(6)$_4$/(18)	$\sum$ (19)→(22)	(14) − (23)	(24) − (9)
	x10⁶	x10⁻⁶	x10⁻⁶	x10⁻⁶	x10⁻⁶	x10⁶	x10⁶	x10⁶	x10⁶	x10⁶	x10⁶	x10⁶
FIRST TRIAL												
1	10,000	.500	--	--	--	2,620	--	--	--	2,620	7,380	6,070
2	10,000	1.500	.500	--	--	870	1,510	--	--	2,380	7,620	5,610
3	10,000	1.500	1.500	.500	--	520	470	280	--	1,620	8,390	6,220
4	10,000	.500	.500	.500	.500	2,620	1,400	290	-960	3,350	6,650	5,010
5	10,000	--	--	--	--	--	--	--	--	--	--	--
SECOND TRIAL												
1	10,000	.500	--	--	--	2,580	--	--	--	2,580	7,420	6,130
2	10,000	1.500	.500	--	--	860	1,360	--	--	2,220	7,780	5,810
3	10,000	1.500	1.500	.500	--	860	450	410	--	1,720	8,280	6,220
4	10,000	.500	.500	.500	.500	2,580	1,360	190	-1050	3,080	6,920	5,380
5	10,000	--	--	--	--	--	--	--	--	--	--	--
THIRD TRIAL												
1	10,000	.500	--	--	--	2,520	--	--	--	2,520	7.480	6,220
2	10,000	1.500	.500	--	--	840	1,280	--	--	2,120	7,88[illegible]	5,980
3	10,000	1.500	1.500	.500	--	840	430	80	--	1,350	8,650	6,710
4	10,000	.500	.500	.500	.500	2,520	1,280	80	-1260	2,620	7,380	6,070
5	10,000	--	--	--	--	--	--	--	--	--	--	--
FOURTH (FINAL) TRIAL												
1	10,000	.500	--	--	--	2,510	--	--	--	2,510	7,490	6,234
2	10,000	1.500	.500	--	--	837	1,265	--	--	2,102	7,898	6,009
3	10,000	1.500	1.500	.500	--	837	422	64	--	1,343	8,657	6,736
4	10,000	.500	.500	.500	.500	2,510	1,265	64	-1284	2,555	7,445	6,166
5	10,000	--	--	--	--	--	--	--	--	--	--	--

TABLE III.14
RESULTS OBTAINED ASSUMING THE ENTIRE BASE STRUCTURE TO BE EFFECTIVE

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
ELEM.	Δ DIFF. IN STRAIN $\Delta(\delta_s-\delta_D)$	FASTENER SLOP Δc	FASTENER STRAIN $\Delta\delta_F$	FAST. SPR. CONST. k_F	FAST. LOAD P_F	DOUBLER LOAD P_D	DOUBLER SPRING CONST. k_D	DOUBLER STRAIN δ_D	INT. LOADS	ACC. INT. LOADS	ACC. APP. LOADS	LOAD IN BASE STR.	BASE STR. SPRING CONSTANT k_s	BASE STR. STRAIN δ_s	DIFF. IN STRAIN $\delta_s-\delta_D$
n	(2)$_{n-1}$ − (16)$_{n-1}$	DATA	(2) − (3)	DATA	(4) − (5)	$\sum$ (6)	DATA	(7)/(8)	DATA	$\sum$ (10)	q_L + (11)	(12) − (7)	DATA	(13) / (14)	(15) − (9)
	x10⁶	x10⁶	x10⁶	x10⁻⁶			x10⁻⁶	x10⁶					x10⁻⁶	x10⁶	x10⁶
1	15,280	0	15,280	.100	1528	1,528	1.000	1,528	0	0	40,000	38,472	4.000	9,618	8,090
2	7,190	0	7,190	.100	719	2,247	1.000	2,247	0	0	40,000	37,753	4.000	9,438	7,191
3	-1	0	-1	.100	0	2,247	1.000	2,247	0	0	40,000	37,753	4.000	9,438	7,191
4	-7,192	0	-7,192	.100	-719	1,528	1.000	1,528	0	0	40,000	37,753	4.000	9,438	7,191
5	-15,282	0	-15,282	.100	-1528	0	--	--	--	--	--	--	--	--	--

SECTION IV

COMPUTER ROUTINES

IV.1 INTRODUCTION

Because of time and/or the complexity of the doubler or splice a hand analysis may not be feasible. A routine for determining the internal load distribution by computer is then desirable or necessary. One such routine, using a digital computer is presented and discussed in Article IV.2. Another method, using an analog computer is discussed in Article IV.3. Other digital computer routines including one designed for splices with multiple members ("stacked" splices) are mentioned in Art. IV.4. All are based upon what is referred to as the elementary theory in this report.

IV.2 GENERAL ROUTINES FOR ANALYSIS BY DIGITAL COMPUTER

Routines have been established for accomplishing the analyses by digital computer. The routines essentially perform the same operations as shown in Table III.1 and III.2 and their accompanying discussions in Section III. In addition, the routines have been extended to include the effects of fastener (joint) plasticity and to present the residual loads existing after an excursion into the plastic range of the fastener load-deflection curves. The weight of the doubler is also computed. This weight does not allow for the holes or for the weight of the attachments themselves.

The basic input data is the same as for the hand analysis method. However, the computer calculates the spring constants of the axial members, requiring an input only of the width and thickness of the members and the fastener hole diameters. Also, it is not necessary to make the initial "guess" for the end fastener load since this (and subsequent guesses) is made by the computer.

The program for the doubler analyses is presented in Figure IV.1. The computer programs for a splice, for stacked doublers, and for stacked splices along with the data and output are presented in Appendix III. The splice program is almost identical to that for the doubler. The stacked doubler and splice programs are for elastic problems without slop. The other programs include provisions for both "slop" and fastener loads in the plastic range.

The first 34 program lines are format, dimension, integer or double precision statements. Statement 35 reads the number of problems to be worked during the run. Statement 41 reads the problem configuration number and case number. Statement 44 reads if residual loads are desired. A positive number if residual loads are required and zero if not. Statement 45 reads the modulus of elasticity for the base structure and doubler. Statement 46 reads the rows of fasteners in the problem and 47 reads the doubler density.

Statements 48 - 56 are data write statements. Statement 63 reads the average length, width, and thickness of the doubler in front of the first fastener for weight calculations. Statement 64 reads the data for each station and statements 66 - 67 writes the data out. Statements 70 - 71 calculated the base structure and doubler spring constants for each fastener station. Statements 76 - 78 calculates the doubler weight.

Statement 79 reads the axial load on the base structure. Statements 92 - 97 reads and writes the fastener spring constants and "cut-off" and allowable load data for the specific spring constants. Statements 110 - 152 change the fastener spring constant if the "cut-off" or allowable load for the specific spring constant used is exceeded. A fastener load-deflection curve is illustrated by Figure IV.2 which explains the fastener cut-off and allowable loads. The multiple slopes of the load-deflection curve allow an accurate fastener spring constant definition to be used. If desired, less than six slopes can be used.

Statements 155 - 157 calculate the first fastener load guess. Statements 159 - 222 change the first guess fastener load to a number nearer the actual fastener load. If the problem has a sloppy first fastener, the second fastener load is adjusted. If the load is increased until the slop closes up in the first fastener, the first fastener load is adjusted for subsequent load increments.

The total load is compared to the doubler load as the doubler load after the last fastener. The doubler load must be within less than 25% of the total applied load after the last fastener. If the doubler load is greater than 25% of the total load (magnitude), the first fastener load is adjusted by + 125 lb. to - 500 lbs. to $1.x10^{-9}$ lbs.

If the first fastener load is adjusted by $1.x10^{-10}$ pounds and the doubler load after the last fastener is not equal to zero, the problem is too sensitive and a solution can not be obtained without combining some of the fasteners into groups as explained in Article III.5 and Figure III.9.

Statements 224 thru 270 are the first fastener load and calculate the remaining fastener loads, doubler loads, and base structure loads. Within this section, statements 235 - 253 check each fastener station for sloppy fasteners. If slop is found at a station, the fastener load at that station is made equal to zero and the base structure spring constant and the doubler spring constant for the preceding fastener is combined with the spring constants at the sloppy fastener station. Statements 274 thru 279 check the doubler load after the last fastener and if the magnitude is not less than 25% of the applied load the first fastener load is adjusted.

Statements 281 thru 288 adjusts the third point first fastener load if the third point extrapolation does not dictate a doubler load of zero after the last fastener.

Statements 289 thru 320 involves making a second guess based on the first point. After the second guess first fastener load is obtained the doubler load, base structure load, and the remaining fastener loads are calculated.

The statements 321 thru 409 calculates the third set of data points based upon the first two sets of points. The extrapolation, statement 393, is method used to "zero in" on the correct fastener loads. The terms of this equation are double precision, sixteen significant digits, to allow the needed accuracy for the first fastener load extrapolation. If the third point extrapolation does not "zero in" on the correct load, statement 403 thru 407 sends the problem back to statements 281 thru 288 to make the needed adjustment. Within this third point calculation are statements 343 thru 360 which checks to see if slop is taken out of the problem and statements 370 thru 388 to see if the fastener cut-off load or allowable is exceeded.

Statements 430 thru 432 calculates the slop remaining at any fastener as the doubler is loaded.

Statements 446 thru 464 keeps a record of the loads and totals the loads as the doubler is loaded. If the fastener cut off load is exceeded the spring constant is changed for that fastener. If any fastener cut-off load is exceeded or slop removed, the same process is repeated with the changed spring constants and the remaining loads until the total load is carried by the base structure and doubler, and the fastener cut-off loads or the allowable loads are not exceeded. If the fastener allowable is exceeded the problem goes to 481 thru 483 where the fastener, the failed, and the total load at failure is recorded.

Statement 491 writes the load data at each station after the problem is complete. Statements 497 and 499 writes the doubler weight. Statement 500 checks to see if residual loads are required. Statement 502 checks to see if all of the problem sets are complete.

Every program follows the basic format of establishing two data points and solving for the third correct point. Example input and example output data is shown on the following pages in Figure IV.3 and IV.4 respectively.

The data for the plastic doubler and splice computer is explained in Appendix IV along with the stacked splice and doubler data.

```
C     PLASTIC  DOUBLER
      279 FORMAT(//1X,37HFIRST FASTENER FAILURE AND TOTAL LOAD//)
      459 FORMAT(3X,2HXL,5X,3HXDI,3X,3HXWC,3X,3HXLU,5X,3HXTS,3X,3HXWS,4X,
         X2HXS,2X,3HXNR,2X,3HXCO)
      462 FORMAT(//1X,4HXQI=,F7.0)
      451 FORMAT(//1X,13HCONFIGURATION,1X,4HNO.=,I10)
      452 FORMAT(1X,4HCASE,1X,4HNO.=I10)
      457 FORMAT(1X,3HXN=,F6.0)
      454 FORMAT(/1X,4HPLA=,F6.0)
      455 FORMAT(1X,4HXED=,F9.0)
      456 FORMAT(1X,4HXES=,F9.0)
      438 FORMAT(1X,3HXW=,F6.4)
      897 FORMAT(F10.2)
      461 FORMAT(1H1,1X,8HXAL(I,1),2X,8HXAL(I,2),2X,8HXAL(I,3),2X,8HXAL(I,4)
         1,2X,8HXAL(I,5),2X,8HXAL(I,6))
      460 FORMAT(1H1,1X,8HXKA(I,1),3X,8HXKA(I,2),3X,8HXKA(I,3),3X,8HXKA(I,4)
         1,3X,8HXKA(I,5),3X,8HXKA(I,6))
      453 FORMAT(1H1,20X,7HDOUBLER,1X,5HINPUT)
      450 FORMAT(2I10)
       27 FORMAT(F13.3)
       28 FORMAT(//3X,7HDOUBLER,2X,6HWEIGHT)
       29 FORMAT(F6.4)
       17 FORMAT(34X,7HDOUBLER,1X,3HANS/)
       14 FORMAT(F6.0)
       13  FORMAT(F7.0)
      496 FORMAT(1X,3HSAY,1X,6HFELLOW,1H,,4HTHIS,1X,7HPROBLEM,1X,2HIS,1X,
         X3HTOO,1X,9HSENSITIVE,1H,,7HREGROUP,1X,9HFASTENERS)
       19 FORMAT(1X,2HXZ,2X,3HXNR,3X,3HXKA,7X,3HXPA,5X,3HXDL,6X,3HXKD
         1,6X,3HXQT,5[illegible] 3HXQB,8X,3HXKS)
       18 FORMAT(2F1[illegible])
       21 FORMAT(6F10.0)
       20 FORMAT(6F11.0)
       10 FORMAT(8F10.4)
       11 FORMAT(F8.5,F6.3,F6.2,F8.5,F6.3,F6.2,F6.3,F4.0,F7.0)
       16 FORMAT(F4.0,F4.0,F9.0,2F8.0,F11.0,2F8.0,F11.0)
          DIMENSION XKD(99),XKS(99)    ,XKDD(99),XKSS(99),XLSS(99)
          DIMENSION XL(99),XDT(99),XWD(99),XLK(99),XTS(99),XWS(99),
         1XS(99),XNR(99),XQO(99),XLU(99),Z(99),XQK(99)
          DIMENSION XKA(99,6),XD(99),XPF(99),XB(99),XT(99),XTC(99)
          INTEGER XST,XZP,XMC,XO,XTT,XJM,XC,RYT,PLA
           DOUBLE PRECISION   XSD,XAS,XCS,XTDA,XR,XPA,XZA,XZB,XDLA,XDLB,XTD,
         1XQB,XBS,XRP,XDL,XAP(99),XLD(99),XPQ(99),XAL(99,6),XYZ,XP,XPR
         1,XAW(99),XA2(99)  ,XSSP(99)
          READ(5,14) XKP
          NKP=0
          NNP=XKP
      950 CONTINUE
          WT=0.0
          WS=0.0
          READ(5,450) AA,A[illegible]
          NKP=NKP+1
```

Figure IV.1. Doubler Program

```
S.0043          RYT=0
S.0044          READ(5,14) PLA
S.0045          READ(5,18) XED,XES
S.0046           READ(5,14) XN
S.0047          READ(5,29) XW
S.0048          WRITE(6,453)
S.0049          WRITE(6,451) AA
S.0050          WRITE(6,452) AB
S.0051          WRITE(6,454) PLA
S.0052          WRITE(6,455)XED
S.0053          WRITE(6,456) XES
S.0054          N=XN
S.0055          WRITE(6,457) XN
S.0056          WRITE(6,438) XW
S.0057          XLRP=1.0
S.0058          DO 100 I=1,N
S.0059          XAW(I)=0.0
S.0060      100 Z(I)=1.
S.0061          NT=N-1
S.0062          XWT=0
S.0063          READ(5,10) XDTA,XWDA,XLLA
S.0064          READ(5,10)(XL(I),XDT(I),XWD(I),XLU(I),XTS(I),XWS(I),
               XI=1,N)                               XS(I),XNR(I),
S.0065          READ(5,897) (XQC(I),I=1,N)
S.0066          WRITE(6,459)
S.0067          WRITE(6,11) (XL(I),XDT(I),XWD(I),XLU(I),XTS(I),XWS(I)
               1XNR(I),XQC(I),I=1,N)                         ,XS(I),
S.0068          DO 195 I=1,N
S.0069          XQK(I)=0.0
S.0070          XKD(I)=XDT(I)*XWD(I)*XED/XLU(I)
S.0071          XKS(I)=XTS(I)*XWS(I)*XES/XLU(I)
S.0072          XKSS(I)=XKS(I)
S.0073          XKDD(I)=XKD(I)
S.0074          XAW(I)=0.0
S.0075           XLSS(I)=XS(I)
S.0076      195 XWT=XLU(I)*XWD(I)*XDT(I)*XW+XWT
S.0077          XKT=XLUA*XWDA*XDTA
S.0078          XWT=XWT+XKT*XW
S.0079          READ(5,13) XQP
S.0080          XQI=XQP
S.0081          XTQ(N)=0.0
S.0082          GO TO 979
S.0083      970 CONTINUE
S.0084          RYT=1.
S.0085          XQI=-XTQ(I)+XTQ(I)/XYR*XQQK
S.0086          DO 1055 I=1,N
S.0087          XQC(I)=-XQK(I)
S.0088          XS(I)=XLSS(I)
S.0089     1055 CONTINUE
S.0090          PLA=0.0
S.0091      979 CONTINUE
```

Figure IV.1. Doubler Program (Continued)

```
S.0092        READ(5,20)(XKA(I,1), XKA(I,2),XKA(I,3),XKA(I,4),XKA(I,5),XKA(I,6)
             1,I=1,N)
S.0093        READ(5,21)(XAL(I,1),XAL(I,2),XAL(I,3),XAL(I,4),XAL(I,5),XAL(I,6)
             1,I=1,N)
S.0094        WRITE(6,460)
S.0095        WRITE(6,20)(XKA(I,1),XKA(I,2),XKA(I,3),XKA(I,4),XKA(I,5),
             1XKA(I,6),I=1,N)
S.0096        WRITE(6,461)
S.0097        WRITE(6,21)(XAL(I,1),XAL(I,2),XAL(I,3),XAL(I,4),XAL(I,5),
             1XAL(I,6),I=1,N)
S.0098        WRITE(6,462) XQP
S.0099        XZP=0
S.0100        XY=0
S.0101        XP=0.D0
S.0102        XAM=1.
S.0103        XTT=-1.
S.0104        XST=0
S.0105        XPR=0
S.0106        XIP=0
S.0107        J=1
S.0108        I=1
S.0109        GO TO 430
S.0110    400 CONTINUE
S.0111        WT=0.0
S.0112        WS=0.0
S.0113        IF(.9999-XP) 302,302,1798
S.0114   1798 CONTINUE
S.0115        IF(XP) 401,1302,401
S.0116   1302 CONTINUE
S.0117        IF(ABS(XQI)-ABS(XQP)) 401,302,401
S.0118    401 CONTINUE
S.0119        DO 1005 I=1,N
S.0120        XQO(I)=XQO(I)*(1.-XP)
S.0121   1005 CONTINUE
S.0122        XQI=XQI*(1.-XP)
S.0123    458 CONTINUE
S.0124        XZP=0
S.0125        XY=0
S.0126        XAM=1.
S.0127        XTT=-1.
S.0128        XST=-1.
S.0129        IF(XUT) 370,430,371
S.0130    371 CONTINUE
S.0131        JJJ=ZK
S.0132        IF(Z(III)-6.) 840,840,998
S.0133    840 CONTINUE
S.0134        IKA=XAL(III,JJJ+1)
S.0135        IF(IKA) 999,995,368
S.0136    368 CONTINUE
S.0137        XKA(III,J   )=XKA(III,JJJ+1)
S.0138        XAL(III,J   )=XAL(III,JJJ+1)
```

Figure IV.1. Doubler Program (Continued)

```
      Z(III)=ZK+1.
      GO TO 370
  995 II=III
      GO TO 998
  370 CONTINUE
      JJ=YK
      Z(II)=YK+1.
      IF(Z(II)-6.) 79,79,998
   79 CONTINUE
      IKS=XAL(II,JJ+1)
      IF(IKS) 999,998,429
  429 CONTINUE
      XAL(II,J )=XAL(II,JJ+1)
      XKA(II,J )=XKA(II,JJ+1)
  430 CONTINUE
      I=1
      XAED=XDT(I)*XWD(I)*XED
      XAES=XTS(I)*XWS(I)*XES
      XPA=((8./XN)/(XAED+XAES))*XGI*XAED
      GO TO 56
   49 IF(XZP) 183,180,181
  181 XAM=.1
      XJM=1.
      XTT=1.
      XPA=XR+XAM
      GO TO 32
  180 XAM=125.
      XPA=XR+XAM
      XTT=0
      GO TO 32
  183 IF(XMC) 186,185,184
  184 XAM=.001
       XPA = XR+XAM
      XJM=0
      GO TO 32
  185 XAM=.00001
      XPA=XR+XAM
      XJM=-1.
      XQ=-1
      GO TO 32
  186 IF(XC) 187,188,189
  187 XAM=.0000001
      XPA=XR+XAM
      XQ=0
      GO TO 32
  188 XAM=.000000001
      XPA=XR+XAM
      XQ=1.
      GO TO 32
  189 CONTINUE
      WRITE(6,496)
```

Figure IV.1 Doubler Program (Continued)

```
      GO TO 999
  51  IF(XTT) 31,34,33
  34  XAM=-5.
      XPA=XR+XAM
      XZP=1.
      GO TO 32
  33  IF(XJM) 37,36,35
  35  XAM=-.01
      XPA=XR+XAM
      XMC=1.
      XZP=-1.
      GO TO 32
  36  XAM=-.0001
      XPA=XR+XAM
      XMC=0
      GO TO 32
  37  IF(XQ) 38,39,40
  38  XAM=-.000001
      XPA=XR+XAM
      XQ=-1.
      XMC=-1.
      GO TO 32
  39  XAM=-.00000001
      XPA=XR+XAM
      XQ=0
      GO TO 32
  40  XAM=-.0000000001
      XPA=XR+XAM
      XQ=1
      GO TO 32
  31  XAM=-500.
      XPA=XR+XAM
  32  XR=XPA
      I=1
  56  XZA=XMR(I)*XPA/XKA(I,J)+XS(I)
      XDS=0
      XDLA=0
      XZ=0
      XQS=0
      XR=XPA
      XTDA=XR
      XTD=XZA
      GO TO 80
  81  CONTINUE
      I=I+1
      XTD=XTD-XDS
  80  CONTINUE
      XAS=XTD
      IF(XS(I+1)) 424,428,424
 424  CONTINUE
      XPA=0.0
```

Figure IV.1. Doubler Program (Continued)

```
      IF(XLPP) 165,165,1001
 1001 CONTINUE
      IF(XZ-XN+1.) 561,165,165
  561 CONTINUE
      XKDD(I)=XKD(I)
      XKSS(I)=XKS(I)
      GO TO 165
  428 CONTINUE
      IF(I-1) 999,426,425
  425 CONTINUE
      IF(XS(I)) 427,426,427
  427 CONTINUE
      XKDD(I)=XKD(I)
      XKSS(I)=XKS(I)
  426 CONTINUE
      XPA=XAS*XKA(I,J)
  165 CONTINUE
      XDLA=XDLA+XPA*XNR(I)
      XSD=XLDA/XKDD(I)
      XQS=XL(I)*XQP(I)+XQS
      XQT=XQS+XQI
      XQB=XQT-XDLA
      XBS=XQB/XKSS(I)
      XDS=XBS-XSD
      XZ=XZ+1.
      IF(XST) 589,598,589
  598 XYR=XQS+XQP
      XQQK=XQS
  589 CONTINUE
      IF(XN-XZ)   101,101,81
  101 CONTINUE
      IF(XQT) 233,53,238
  233 CONTINUE
      IF(XDLA/XQT-.25)  42,42,49
   42 IF(.25-XDLA/XQT) 51,53,53
  238 CONTINUE
      IF(XDLA-.25*XQT) 57,57,51
   57 IF(.25*XQT+XDLA) 49,53,53
   53 CONTINUE
      GO TO 71
   88 CONTINUE
      XZA=XTDA
      XDLA=XLD(I)
      XR=XRP*XKA(I,J)
      I=1
      XZB=XZA+XDLA*(XZA-XZB)/(XCLB-XDLA)
      XPA=XKA(I,J)*(XZB-XS(I))
       IF(XZB-XZA) 95,999,95
   71 I=1
      XPA=XR+XAM/10.
  124 XZB=XNR(I)*XPA/XKA(1,J)+XS(I)
```

Figure IV.1. Doubler Program (Continued)

```
   95 XTD=XZB
      XR=XPA
      XDS=0
      XDLB=0
      XZ=0
      XQS=0
      GO TO 84
   85 CONTINUE
      I=I+1
   84 XTD=XTD-XDS
      XAS=XTD
      IF(XS(I)) 419,418,419
  419 CONTINUE
      XSSP(I)=XTD
      XPA=0.0
      GO TO 265
  418 CONTINUE
      XPA=XAS*XKA(I,J)
  265 CONTINUE
      XDLB=XDLB+XPA*XNR(I)
      XSD=XDLB/XKDD(I)
      XQS=XL(I)*XQC(I)+XQS
      XQT=XQS+XQI
      XQB=XQT-XDLB
      XBS=XQB/XKSS(I)
      XDS=XBS-XSD
      XZ=XZ+1.
      IF(XN-XZ) 103,103,85
  103 CONTINUE
   87 CONTINUE
      XPR=0
      XZ=0
      I=1
      XLD(I)=0
      XQS=0
      XDS=0
      XY=0
      XPI=0
      XVF=XP
      XUT=0.0
      XP=0.0
  131 XTD=XZB+XDLB*(XZB-XZA)/(XDLA-XDLB)
      XTDA=XTD
  132 XRP=XTDA
      GO TO 86
   74 CONTINUE
      I=I+1
      XTD=XTD-XDS
   86 CONTINUE
      XAS=XTD
      IF(XS(I)) 409,408,409
```

Figure IV.1. Doubler Program (Continued)

```
  409 CONTINUE
      XAP(I)=0.0
      XSSP(I)=XTD
      WT=(DABS(XTD)-XS(I))/DABS(XTD)
      IF(WT) 389,390,390
  389 CONTINUE
      WT=0.0
      GO TO 332
  390 CONTINUE
      WT=ABS(WT)
      IF(WT-XP) 332,374,375
  375 XP=WT
      II=I
      GO TO 332
  374 CONTINUE
      III=I
      GO TO 332
  408 CONTINUE
  348 CONTINUE
       XAP(I)=XAS*XKA(I,J)
      XA2(I)=XTD
  365 CONTINUE
      IF(KYT) 648,648,331
  648 CONTINUE
      IF(XST) 937,909,999
  909 CONTINUE
      XPF(I)=0
  937 CONTINUE
      XYZ=XAL(I,J)-ABS(XPF(I))
      IF(DABS(XYZ)-DABS(XAP(I))) 306,306,331
  306 WT=DABS(XYZ/XAP(I))
      WS=XP
      WT=1.-WT
      IF( WT-WS) 331,309,308
  309 CONTINUE
      III=I
      ZK=Z(I)
      XLT=1.
      GO TO 332
  309 II=I
      YK=Z(I)
      XPI=1.
      XLT=-1.
      XP=DABS(XYZ/ XAP(I))
      XP=1.-XP
      GO TO 332
  331 CONTINUE
  332 IF(I-1) 750,775,750
  775 XLD(I)=XAP(I)*XNR(I)
      GO TO 800
  750 XLD(I)=XLD(I-1)+XAP(I)*XNR(I)
```

Figure IV.1. Doubler Program (Continued)

```
  800 CONTINUE
      XSD=XLD(I)/XKDD(I)
      XQS=XL(I)*XQ0(I)+XQS
      XQT=XQS+XQI
      XBQ(I)=XQT-XLD(I)
      XBS=XBQ(I)/XKSS(I)
      XDS=XBS-XSD
      XZ=XZ+1.
  117 IF(XN-XZ) 102,102,74
  102 CONTINUE
      AXLD=XLD(I)
      AAQT=.0001*XQT
      IF( ABS(AXLD)- ABS(AAQT  )) 880,880,88
  880 CONTINUE
      IF(XS(II)*1000.) 481,421,481
  481 CONTINUE
      XLT=0.0
      XSSP(II)=0.0
      XS(II)=0
      XKDD(II)=XKD(II)
      XKSS(II)=XKS(II)
      IF(II-1) 479,421,479
  479 CONTINUE
      XKDD(II-1)=XKD(II-1)
      XKSS(II-1)=XKS(II-1)
  421 CONTINUE
      IF(XS(III)*1000.)  515,422,515
  515 CONTINUE
      XS(III)=0.0
      XSSP(III)=0.0
      XKDD(III)=XKD(III)
      XKSS(III)=XKS(III)
      XKDD(III-1)=XKD(III-1)
      XKSS(III-1)=XKS(III-1)
  422 CONTINUE
      XP=1.-XP
      DO 1000 I=1,N
      XS(I)=XS(I)-DABS(XSSP(I)*XP)
 1000 CONTINUE
      IF(RYT) 70,70,359
   70 CONTINUE
      IF(XP) 359,300,359
  300 XP=1.
  359 CONTINUE
      I=1
      XZ=1.
      IF(XST) 737,707,999
  707 CONTINUE
      IF(RYT) 708,708,737
  708 CONTINUE
      GO TO 736
```

Figure IV.1. Doubler Program (Continued)

```
  735 I=I+1
  736 CONTINUE
      XB(I)=0
      XD(I)=0
      XTQ(I)=0
      XPF(I)=0
      IF(N-I) 999,734,735
  734 I=1
      GO TO 737
   65 CONTINUE
      I=I+1
      XZ=XZ+1.
                  XQK(I)=XQO(I)*XP+XQK(I)
  737 CONTINUE
      XPF(I)=XP*XAP(I)+XPF(I)
      XD(I)=XLD(I)*XP+XD(I)
      XB(I)=XBC(I)*XP+XB(I)
      XTQ(I)=(XBQ(I)+XLD(I))*XP+XTQ(I)
      XBQ(I)=XTQ(I)-XD(I)
      XLRP=0.0
      IF(XN-XZ) 301,301,65
  301 CONTINUE
      IF(RYT) 485,485,486
  486 CONTINUE
      ITQ=XTQ(I)
      IF(ITQ) 400,302,400
  485 CONTINUE
      IYR=XYR
      ITQ=XTQ(I)
  711 CONTINUE
      IF(IYR-ITQ) 505,302,400
  505 ABC=IABS(IYR-ITQ)
      IF(ABC-.001*XYR) 302,302,305
  302 I=1
      GO TO 304
  998 CONTINUE
      XI=II
      WRITE(6,279)
      WRITE(6,18) XI,XTQ(I)
      GO TO 302
  303 I=I+1
      XZ=XZ+1.
      GO TO 410
  304  WRITE(6,17)
       WRITE(6,19)
      XZ=1.
  410 CONTINUE
      WRITE(6,16) XZ,XNR(I),XKA(I,J),XPF(I),XD(I),XKD(I)
     1XB(I),XKS(I)                                        ,XTQ(I),
      IF(XN-XZ) 999,999,303
  305 XP=(XYR-XTQ(I))/XYR
      XZ=1.
```

Figure IV.1. Doubler Program (Continued)

```
      I=1
      GO TO 737
  999 CONTINUE
      WRITE(6,28)
      WRITE(6,27) XWT
      IF(PLA) 980,980,970
  980 CONTINUE
      IF(NKP-NNP) 950,951,951
  951 CONTINUE
      STOP
      END
```

Figure IV.1. Doubler Program (Concluded)

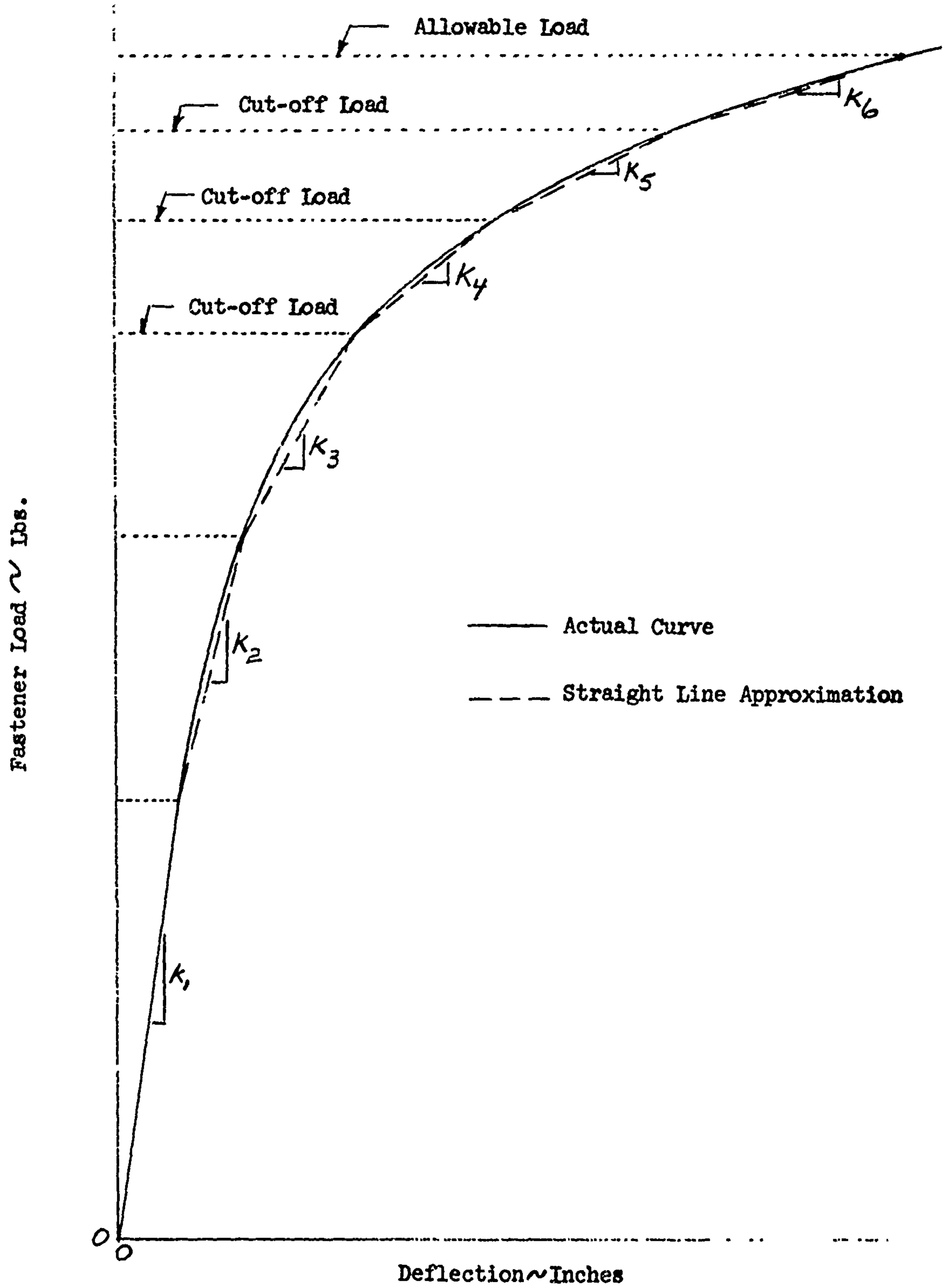


Figure IV.2. Load-Deflection Curve for a Fastened Joint Replaced by Straight Line Increments

```
1.
1          3
1.
103.0000.  1.3....
14.
.1
    .355      1.25      .75
   1.      .172   1.38   1.0   .102   2.88   .0     1.0
   1.0     .072   1.38   1.0   .102   2.88   .1     1.0
   1.0     .172   1.38   1.0   .102   2.88   .001   1.0
   1.0     .072   1.38   1.0   .102   2.88   .001   1.0
   1.0     .072   1.38   1.0   .102   2.88   .001   1.0
   1.0     .072   1.38   1.0   .102   2.88   .001   1.0
   1.0     .072   1.38   1.0   .102   2.88   .001   1.0
   1.0     .072   1.38   1.0   .102   2.88   .0     1.0
   1.0     .072   1.38   1.0   .102   2.88   .0     1.0
   1.0     .072   1.38   1.0   .102   2.88   .001   1.0
   1.0     .072   1.38   1.0   .102   2.88   .001   1.0
   1.      .072   1.38   1.0   .102   2.88   .001   1.0
   1.0     .072   1.38   1.0   .102   2.88   .001   1.0
   1.0     .172   1.38   1.0   .102   2.88   .001   1.0
   1.0     .172   1.38   1.0   .102   2.88   .001   1.0
   1.0     .072   1.38   1.0   .102   2.88   .001   1.0
225.
225.
225.
225.
225.
225.
225.
225.
225.
225.
225.
225.
225.
225.
225.
 0.0
18000.
   117500.   115600.   69700.   32000.   19200.   12900.
   117500.   115600.   69700.   32000.   19200.   12900.
   117500.   115600.   69700.   32000.   19200.   12900.
   117500.   115600.   69700.   32000.   19200.   12900.
   117500.   115600.   69700.   32000.   19200.   12900.
   117500.   115600.   69700.   32000.   19200.   12900.
   117500.   115600.   69700.   32000.   19200.   12900.
   117500.   115600.   69700.   32000.   19200.   12900.
   117500.   115600.   69700.   32000.   19200.   12900.
   117500.   115600.   69700.   32000.   19200.   12900.
   117500.   115600.   69700.   32000.   19200.   12900.
   117500.   115600.   69700.   32000.   19200.   12900.
   117500.   115600.   69700.   32000.   19200.   12900.
```

Figure IV.3. Example Input Data

117500.	105600.	6970.	32000.	19200.	12900.
117500.	105600.	6970.	32000.	19200.	12900.
117500.	105600.	6970.	32000.	19200.	12900.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
117500.					
117500.					
117500.					
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10000.					

Figure IV.3. Example Input Data (Concluded)

OUTPUT DATA

XZ = fastener row

XNR(I) = no. of fasteners in row

XDA(I, J) = spring constant of fastener

XPF(I) = fastener load at I fastener station

XD(I) = doubler load at fastener station I

SKD(I) = doubler spring constant at I

XTQ(I) = total load at Station I

XB(I) = load in base structure at I

XKS(I) = effective base spring constant at I

XWT = weight of doubler

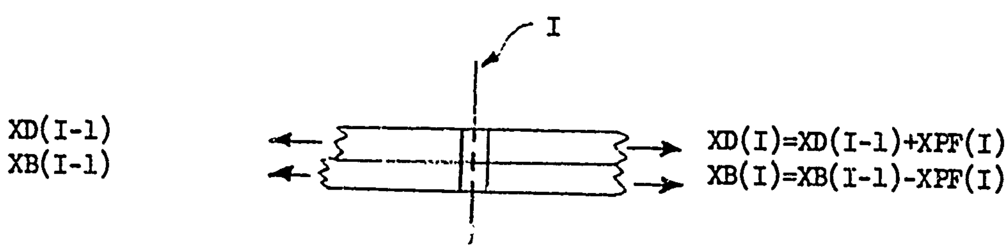


Figure IV.4. Example Output Data

DOUBLER INPUT

CONFIGURATION NO.= 1000000
CASE NO.= 3000000

PLA= 1.
XEC=1030000.
XES=1030000.
XN= 16.
XW=0.1000

XL	XCT	XWC	XLL	XTS	XWS	XS	XNR	XQO
1.00000	0.072	1.38	1.00000	0.102	2.88	0.0	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.100	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.0	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.0	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	225.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.0

XKA(I,1)	XKA(I,2)	XKA(I,3)	XKA(I,4)	XKA(I,5)	XKA(I,6)
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.

Figure IV.4. Example Output Data (Continued)

XAL(I,1)	XAL(I,2)	XAL(I,3)	XAL(I,4)	XAL(I,5)	XAL(I,6)
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.

XCI= 18000.

DOUBLER ANS

XZ	XNR	XKA	XPA	XDL	XKC	XQT	XQB	XKS
1.	1.	32000.	1524.	1524.	1023407.	19225.	16701.	3025726.
2.	1.	117500.	0.	1524.	1023407.	18450.	16926.	3025726.
3.	1.	105600.	1005.	2529.	1023407.	18675.	16146.	3025726.
4.	1.	117500.	697.	3226.	1023407.	13900.	15674.	3025726.
5.	1.	117500.	459.	3685.	1023407.	19125.	15440.	3025726.
6.	1.	117500.	282.	3967.	1023407.	19350.	15382.	3025726.
7.	1.	117500.	152.	4119.	1023407.	19575.	15456.	3025726.
8.	1.	117500.	131.	4250.	1023407.	19800.	15550.	3025726.
9.	1.	117500.	15.	4265.	1023407.	20025.	15760.	3025726.
10.	1.	117500.	0.	4265.	1023407.	20250.	15985.	3025726.
11.	1.	117500.	-121.	4144.	1023407.	20475.	16331.	3025726.
12.	1.	117500.	-280.	3864.	1023407.	20700.	16836.	3025726.
13.	1.	117500.	-490.	3374.	1023407.	20925.	17551.	3025726.
14.	1.	105600.	-781.	2593.	1023407.	21150.	18557.	3025726.
15.	1.	69700.	-1149.	1445.	1023407.	21375.	19930.	3025726.
16.	1.	32000.	-1445.	-0.	1023407.	21375.	21375.	3025726.

DOUBLER WEIGHT
0.164

Figure IV.4. Example Output Data (Continued)

XKA(I,1)	XKA(I,2)	XKA(I,3)	XKA(I,4)	XKA(I,5)	XKA(I,6)
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0
117500.	0.	0.	0.	0.	0.0

XAL(I,1)	XAL(I,2)	XAL(I,3)	XAL(I,4)	XAL(I,5)	XAL(I,6)
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0
10000.	0.	0.	0.	0.	0.0

XCI= 18000.

Figure IV.4. Example Output Data (Continued)

DOUBLER ANS

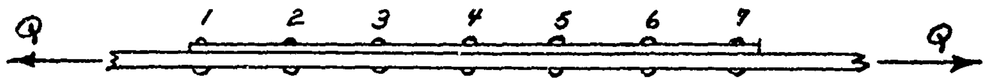
XZ	XNR	XKA	XPA	XDL	XKD	XQT	XQB	XKS
1.	1.	117500.	-353.	-353.	1023407.	-0.	353.	3025726.
2.	1.	117500.	0.	-353.	1023407.	-0.	353.	3025726.
3.	1.	117500.	93.	-260.	1023407.	-0.	260.	3025726.
4.	1.	117500.	81.	-179.	1023407.	-0.	179.	3025726.
5.	1.	117500.	54.	-125.	1023407.	-0.	125.	3025726.
6.	1.	117500.	35.	-90.	1023407.	-0.	90.	3025726.
7.	1.	117500.	21.	-69.	1023407.	-0.	69.	3025726.
8.	1.	117500.	11.	-58.	1023407.	0.	58.	3025726.
9.	1.	117500.	2.	-57.	1023407.	-0.	57.	3025726.
10.	1.	117500.	0.	-57.	1023407.	-0.	57.	3025726.
11.	1.	117500.	-16.	-72.	1023407.	-0.	72.	3025726.
12.	1.	117500.	-27.	-99.	1023407.	0.	99.	3025726.
13.	1.	117500.	-42.	-142.	1023407.	-0.	142.	3025726.
14.	1.	117500.	-60.	-202.	1023407.	-0.	202.	3025726.
15.	1.	117500.	-37.	-239.	1023407.	0.	239.	3025726.
16.	1.	117500.	239.	-0.	1023407.	0.	0.	3025726.

DOUBLER WEIGHT
0.164
STOP 00000

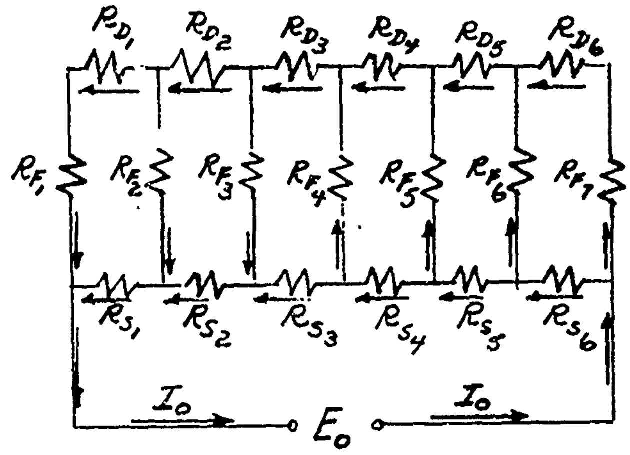
Figure IV.4. Example Output Data (Concluded)

IV.3 ANALOG COMPUTER ANALYSIS

A method of determining the distribution of fastener loads in a splice by using an analog computer is described in detail in Reference (4) and can also be used for a doubler installation. The method consists of replacing the actual structural elements (fasteners and axial members) by an electrical network of resistors in the form of potentiometers. The resistances are adjusted so that the relative values of their reciprocals (or "mhos") are the same as the relative values of the spring constants in the actual structure. That is, $\frac{R_{n+1}}{R_n} = \frac{k_n}{k_{n+1}}$. This is illustrated in Fig. IV.5.



Physical Structure And Applied Load
(a)



Equivalent Electrical Circuit And Applied Current, I_o
(b)

Figure IV.5. A Doubler Installation Analyzed By An Analog Computer

A voltage E_o is applied, generating a total current I_o. The current I_o divides among the resistances in the same manner (proportionally) as the load Q is distributed in the structural network. Therefore, reading I_o and I_n with an ammeter (or by other determination), the load in any structural member can easily be calculated as

$$P_n = Q \times \frac{I_n}{I_o}$$

The analog computer can also be used for multiple (or "stacked") doublers and splices as well as for shear-lag problems in sheet-stringer panels. It can be used for load levels where the values of k_n' are in the plastic range, by using the method of superposition as discussed in Section III. In this case, the resistors would be adjusted for the specific spring constant values existing (as selected per Figure IV.10) for each increment of applied load. Reference (4) also describes a practical constant voltage source necessary for applying a distributed load (i.e., such as an applied shear flow) or any intermediate load. In any case, the same results would be obtained as by using the other methods discussed in Sections III and IV, since they are all derived from the same elementary theory.

IV.4 OTHER DIGITAL COMPUTER PROGRAMS

Although this report is based upon the trial and error solution for the internal loads, the loads can be determined in the conventional manner for redundant structures by solving a set of simultaneous equations. That is, if there are N fasteners in a line in the direction of the applied load, there are N-1 redundant fastener loads. A set of equations can be written for any given condition of the structure (i.e., for any specific values of k_F, k_D, k_s and for any slop, **meaning that the sloppy** fastener is ineffective). Then the results obtained after solving the simultaneous equations can be used as the "unit solutions" discussed in this report. This procedure is frequently used where digital computers are available.

Reference (5) presents a routine for determining the fastener load distributions in splices involving two or more axial members. The basic approach involves the solution of simultaneous equations (hence it is not useful for hand analysis.) Provision is made for including the effects of plasticity and temperature. The method is based on what is referred to as the elementary theory in this report. As presented, however, the routine is not arranged for the analysis of a doubler installation and provision is not made for the inclusion of "slop". Considerable practical discussion concerning the development , use and presentation of fastener load-deflection data is presented and specific data for one type of fastener (Blind Hi-Shear bolts) are included.

IV.5 ADDITIONAL PROGRAMS PRESENTED IN APPENDIX III

Digital computer programs for a splice, a stacked doubler (one extra doubler) and a stacked splice (one extra member) are presented in Appendix III.

SECTION V

DATA FOR ANALYSES

V.1 INTRODUCTION

As discussed in previous sections, there are three specific types of data that are necessary for determining the fastener load distribution. These are

a. The fastener spring constants, k_F

b. The axial member spring constants, k_D and k_S

c. The fastener hole clearance or "slop", Δc

Each of these is discussed below from the standpoint of practical design and analysis.

V.2 FASTENER SPRING CONSTANTS

This factor is the index of the amount of load, ΔP_F , required to strain the joint through a small displacement $\Delta\delta$. The displacement δ (called the "deflection") is the local "shearing" displacement, normal to the centerline of the fastener as shown in Figure V.1. δ is obtained experimentally as the difference between the unloaded length L (actually 2") between points A and B and the stretched length, L + δ, between the points A' and B' under a load P. This deflection, therefore, includes not only the shearing bearing and bending displacements of the fastener but also those due to the local bearing and axial deformations of the sheets in the region of the hole.

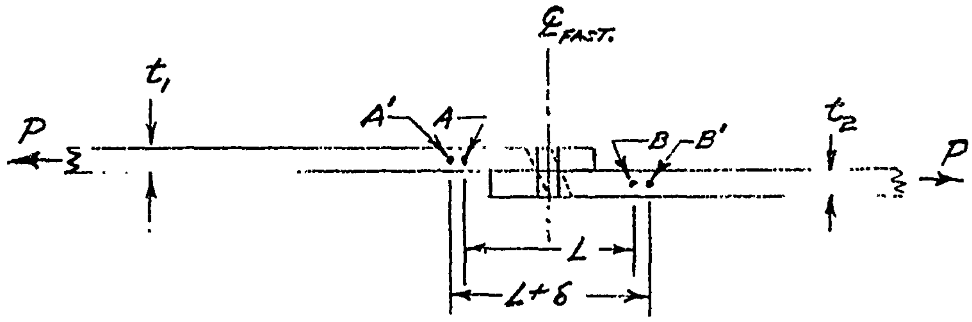


Figure V.1 Deflection at a Joint

By testing specimens as shown in Figure V.1 (which are the same specimens as used in obtaining conventional fastener-sheet strength and yield data) a load-deflection curve for any specific type of joint can be obtained. Such a curve is sketched in Figure V.2. A discussion of the manner in which such a curve is obtained is presented in Section VII.

Frequently the curve has a considerable linear portion at low load levels. The slope of the curve at any point is the value of $k_F = \Delta P/\Delta\delta$. Hence, it can be seen that k_F is a function of the load itself. Thus, k_F is

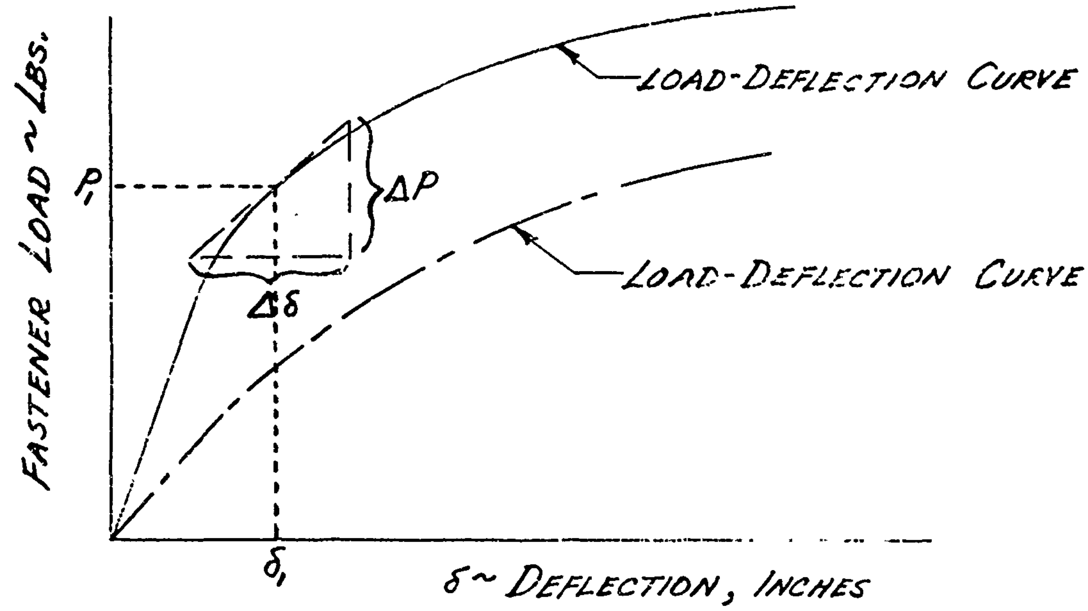


Figure V.2 Typical Load-Deflection Curves for Fastened Joints

analogous to the tangent modulus, E_t , of a stress strain curve for a material. The non-linear portion of the deflection curve is referred to as the "plastic range". In this range k_F decreases from its initial largest value to lesser ones as the value of P increases.

For most of the fasteners and gages used in a practical doubler or splice installation (high strength steel fasteners), there is usually a fairly extensive initial linear portion. This allows the joint to handle reasonable load transfers without excessive permanent set, or yielding.

The exact shape of the load-deflection curve depends upon several items:

a. The fastener type, size, and material properties
b. The material properties of each sheet
c. The thickness of each sheet (different thicknesses giving different results)
d. The fastener hole-clearance or "slop",
e. The number and arrangement of the axial members

Items (a) and (b) are fairly obvious effects. Countersunk types will be more flexible that protruding heads, solid fasteners stiffer than hollow ones (blind types), temperature is a variable since it affects material properties, etc.

As to item (c), most test data appears to be obtained using sheet specimens of the same material and thickness. Hence, when members of significantly different thicknesses (or materials) are

joined either the test data for this particular combination must be obtained experimentally or some reasonable adjustment of available data for other combinations must be made. Although not substantiated by significant testing, the following adjustment is suggested for such cases, referring to Figure V.3.

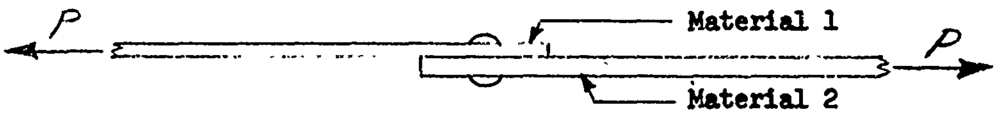


Figure V.3 A Lap Joint Having Dis-similar Sheets

Let k_1 be the value for two members of material and thickness 1.
Let k_2 be the value for two members of material and thickness 2.
Then the "effective" value of k_F for the joint is taken as

$$k_{F_{eff.}} = \frac{2(k_1 k_2)}{k_1 + k_2}$$

As to item (d), a tight hole, or one with little clearance (slop), will result in a stiffer joint that one having a considerable clearance even after the initial clearance has been "closed up" under load. The effect of slop on the load-deflection curve is discussed in Article VII.7.

The number and arrangement of the members will affect the spring constant since these affect the "end fixity" for the fastener. That is, the spring constant is a value relative to two adjacent members and is easily determined by tests as previously discussed for single lap members, or for single sandwich joints (since a sandwich joint is considered in analysis as a single lap joint). However, when the members are stacked, as in Figure III.14a, the relative fixity between adjacent members actually depends upon the loads in all of the members. Hence in this case even an experimental determination of the relative spring contant (i.e., the load-deflection curve) between the adjacent members is a difficult undertaking. This is because each load deflection curve would depend upon the actual test load applied to each member. In addition, the relative deflections between all adjacent members would have to be determined experimentally in order to describe the proper curve for adjacent members. It may be that there is little difference in such spring constants due to variation in member loads, but this subject is not investigated in this report.

Thus, the load deflection curve shown by the broken line in Figure V.2 could be the result, (compared to the solid line) if a less stiff fastener, or sheet material, or a thinner sheet gage were used, or if more "slop" were originally present at the hole. Hence, it can be seen that in order to analyze joints in general, a large amount of load-deflection data defining the fastener spring constants is needed.

Such data are, apparently, not available in the literature at present. This indicates a significant area of technology that needs to be explored to provide the designer with practical data necessary for joint analyses. Very likely, many data of this type are available from various sources, but they are not, unfortunately, in published form. Once determined, such data could be presented in compact tabular form, eliminating the voluminous load-deflection curves. That is, since the load-deflection curves are similar in form and effect to typical material stress-strain curves, it would appear to be advantageous to use the Ramberg-Osgood approach for presenting such fastener data. In this way the actual load-deflection curve for a given fastener sheet combination could be expressed in terms of three parameters, including the shape factor, n. Such a presentation has actually been suggested in some detail in Reference (5) and suggests using the initial slope, k_{F_0}, the yield load, P_Y, and a shape factor, n. This appears to merit consideration, since one table could describe a multitude of practical test data.

For the present, since no sources of general load-deflection data can be referenced, the designer or analyst must determine the spring constants of the fasteners being considered, using whatever data and means he has available. For the particular case of bolts in double shear, References (6) and (7) present a method that will define the bolt spring constant in the elastic range. A few fastener load-deflection curves are also presented in Section VII for the specimens tested in this program.

V.3 AXIAL MEMBER SPRING CONSTANTS

In general it is suggested that these be calculated as

$$k_D = \frac{A_{D_e} E_D}{L}$$

and

$$k_S = \frac{A_{S_e} E_S}{L}$$

where L = length of segment being used (normally the fastener spacing)

A_e = the average cross-sectional area of the element arbitrarily omitting 80% of the diameter of a fastener, in computing this, as being ineffective area. The figure 80% is arbitrary but is the amount used in the calculations of this report. The closer the holes, the more this figure approaches 100% of the fastener diameter. 80% would be more likely to be reasonable for a very close spacing, say 4D or less. The data of Section VII was not sufficient to define this percentage.

E = the tangent modulus (or Young's Modulus in the elastic range)

This calculation is illustrated in Figure V.4.

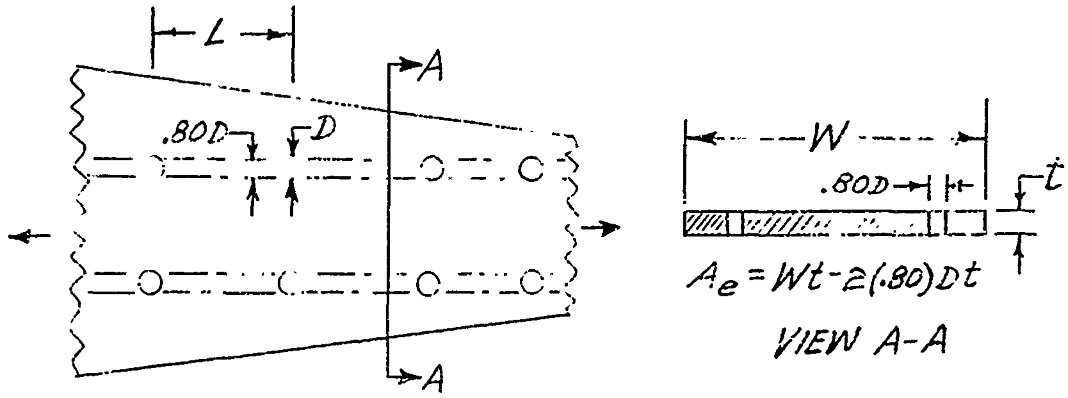


Figure V.4 Effective Area of a Cross Section

If the fasteners have been grouped together, as discussed in Section III, the length, L, is taken as the distance between the centroid of the groups (see Figure III.9c). The area, A_e, however, should be adjusted to reasonably account for the holes, as they actually exist. The adjustment becomes even more arbitrary when the successive holes are not in line.

V.4 FASTENER-HOLE CLEARANCE OR "SLOP"

In this report, the "slop", ΔC, at a fastened joint is defined as the distance over which either sheet can move relative to the other before the fastener bears upon both sheets. This is probably easiest to define by considering the fastener to be fixed in space and then determining the distances over which each sheet can move before bearing upon the fastener. The "slop" will then be the sum of these movements. Referring to Figure V.5 it can be seen that

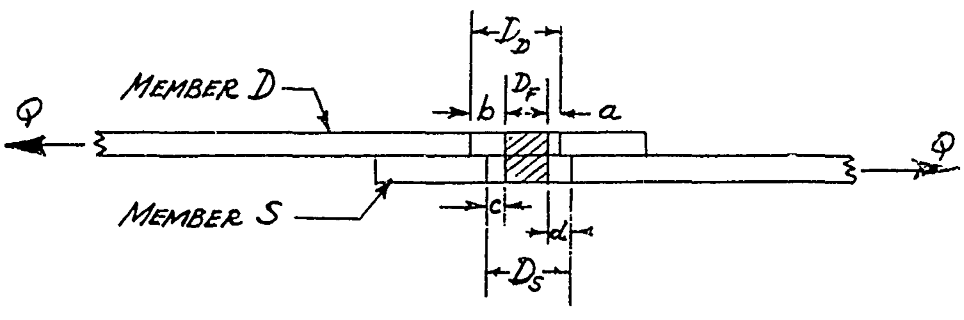


Figure V. 5 "Slop" at a Fastened Joint

for the direction of loading, Q, shown

a. The upper sheet, D, can move a distance "a" before it bears on the fastener (which has the diameter D_F).

b. The lower sheet, S, can move a distance, "c", before bearing on the fastener.
c. Hence the slop at the joint is $\Delta c = a + c$.

If the direction of loading were reversed,

a. The sheet D could move a distance, b
b. The sheet, S, could move a distance, d
c. The slop would then be

$$\Delta c = b + d$$

Thus, it is seen that, in general, the slop depends not only upon the geometry at the joint but also upon the direction of loading. As will be seen later, in the more common case of concentric holes, the direction of loading is not a factor.

A general expression defining the slop in terms of the fastener diamet, hole diameters, hole eccentricities, and direction of loading at the joint can be obtained from Figure V.6.

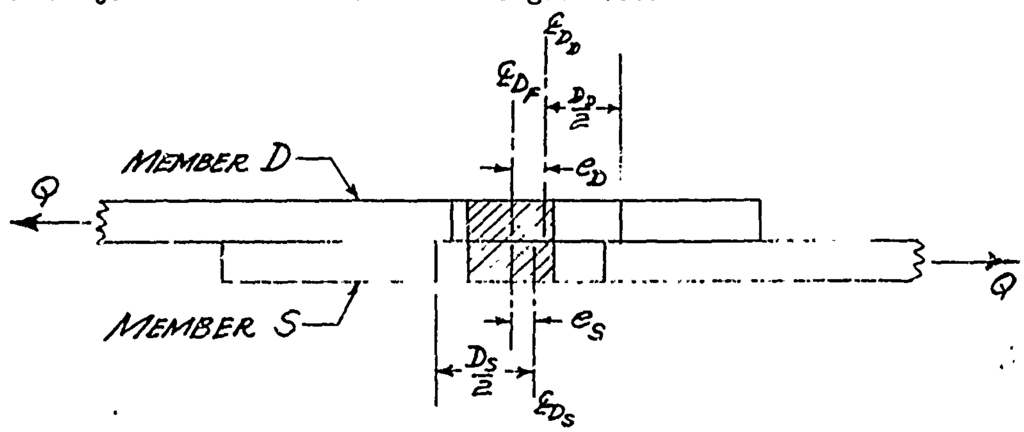


Figure V.6 Slop at a Fastened Joint

D_F = diameter of fastener

D_D = diameter of hole in upper number, D

D_S = diameter of hole in lower number, S

$\mathcal{C}_L$ = center line of fastener or holes

e_D = distance which $\mathcal{C}$ of D_D lies <u>to the right</u> of the of the fastener $\mathcal{C}$,

e_S = distance which $\mathcal{C}_L$ of D_S lies <u>to the right</u> of the of the fastener $\mathcal{C}$.

a. For a "tension" loading, as shown in Figure V.6,

(1) Member D can move a distance $D_D/2 + e_D - D_F/2$ before bearing on the fastener.

(2) Member S can move a distance $D_S/2 - e_S - D_F/2$ before bearing on the fastener.

(3) Hence the slop is the sum of these distances, or

$$\Delta c = \frac{D_D + D_S}{2} - D_F + (e_D - e_S)$$

b. For a reversed loading, producing compressive stresses in the sheets of Figure V.6,

(1) Member D can move a distance $D_D/2 - e_D - D_F/2$ before bearing on the fastener.

(2) Member S can move a distance $D_S/2 + e_S - D_F/2$ before bearing on the fastener.

(3) Hence the slop is the sum of these distances, or

$$\Delta c = \frac{D_D + D_S}{2} - D_F - (e_D - e_S)$$

Thus, it is seen that in one case, tension, the term $(e_D - e_S)$ is added and in the reversed case it is subtracted to obtain the total slop.

In most practical cases the holes will be concentric, or $e_D = e_S$, and

$$\Delta c = \frac{D_D + D_S}{2} - D_F$$

Thus, the slop is independent of the direction of loading. If, as frequently occurs, $D_D = D_S$ $(= D_{hole})$ the slop is simply

$$\Delta c = D_{hole} - D_F$$

The amount of slop to be considered at a joint in any specific structure depends, of course, upon the specified type of fit, the manufacturing and assembly methods and, hence, upon the laws of probability. Thus the determination of the actual amount of slop to be used (except for the salvage of inspected pieces of hardware) is somewhat arbitrary and involves the judgment of the engineer. Hence, it is beyond the scope of this report. In general the following guides are helpful:

a. When a fastener is "sloppy" those fasteners immediately adjacent to it (on each side) pick up more load, than when it is "tight".

b. Slop at the fasteners makes a doubler less efficient. That is, the doubler picks up less load from the base structure it is relieving.

c. The effect of slop at a fastener is much more pronounced in "short members" having only a few fasteners (or rows of fasteners) than in a long member having many fasteners in the direction of the load. Splices are the most usual cases of such "short" members.

d. An analysis which includes the possible or the probable slop is frequently helpful in establishing the type of fit necessary for an assembly.

e. An analysis which includes the existing slop in a specific case is helpful in establishing the course of action necessary in a salvage operation involving sloppy holes.

V.5 EFFECT OF FRICTION

Since in practical cases nearly all fasteners are installed with some amount of "clamp-up", there will always be some accompanying amount of friction force opposing the deflection. This effect can be seen in the actual test data curves of Figures VII.9 and VII.10 as line OA. However, this effect, the initial extra stiffness, is removed in presenting the final load-deflection curves (Figures VII.11 through VII.17) as discussed in Section VII. Hence, friction is ignored.

SECTION VI

APPLICATION OF RESULTS OF ANALYSES TO THE OVERALL STRUCTURE

VI.1 INTRODUCTION

The methods of determining the internal load distributions in splices and doublers are used to properly design such installations. Once installed, these members become an integral part of the overall structure and will influence the distribution of internal loads not only where they are located but also in other areas of the structure. That is, the basic structure has been altered and it is sometimes desirable, or necessary, to include this new effective area in a revised general analysis.

VI.2 PROCEDURE

This can be done for common engineering purposes by determining the "effective" areas of the doubler, or splice members, and including these in any revised overall internal loads analysis. The effective area of the doubler can then be taken (at any station) as

$$A_{eff} = A_{actual} \times \frac{P}{P_o}$$

where

P = Load in doubler from the original analysis (Section III or IV)

P_o = Load that would exist in doubler if it were fully effective with the base structure, or

$$P_o = \text{Applied Axial Load} \times \frac{A_{doubler}}{A_{doubler} + A_{base\ str.}} = Q_L + \sum^{n} q_a L_n$$

Once the effective areas of the doubler are determined, the overall structure can be re-analyzed using conventional methods of analysis. In order to do this the doubler is assigned effective widths at stations along its length that correspond to the effective areas determined (i.e., $W_{eff} = A_{eff} / t$). This effective member is then assumed to be an integral part of the overall structure and future analyses are carried out on this basis, using conventional methods.

VI.3 APPLICATION OF THE RESULTS OF A DOUBLER ANALYSIS

Example

The doubler of Table III.1 would be dealt with as illustrated in Table VI.1 in establishing it as an effective integral part of the base structure.

TABLE VI.1

DETERMINATION OF THE EFFECTIVE AREA AND EFFECTIVE WIDTH OF A DOUBLER

①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫
STA.	A_DE_D	E_D	t_D	A_SE_S	$A_DE_D + A_SE_S$	APPL. LOAD	P_o	P_D	EFFECT. A_DE_D	EFFECT. AREA	EFFECT. WIDTH
	TABLE III.1	DATA	DATA	TABLE III.1	② + ⑤	TABLE III.1	$\frac{②}{⑥}$ × ⑦	TABLE III.1	$\frac{⑨}{⑧}$ x ②	$\frac{⑩}{③}$	$\frac{⑪}{④}$
	$x10^{-6}$	$x10^{-6}$		$x10^{-6}$	$x10^{-6}$				$x10^{-6}$		
1	4.7	29	.10	4.7	9.4	8,000	4,000	385	.45	.016	.16
2	"	"	"	"	"	"	"	1,938	2.28	.079	.79
3	"	"	"	"	"	"	"	2,792	3.28	.113	1.13
4	"	"	"	"	"	"	"	3,234	3.80	.131	1.31
5	"	"	"	"	"	"	"	3,416	4.01	.138	1.38
6	"	"	"	"	"	"	"	3,398	3.99	.138	1.38
7	"	"	"	"	"	"	"	3,176	3.73	.129	1.29
8	"	"	"	"	"	"	"	2,675	3.14	.108	1.08
9	"	"	"	"	"	"	"	1,722	2.03	.070	.70

The desired results, the effective area or the effective width of the doubler, are shown in Columns ⑪ and ⑫ respectively, at the stations listed.

VI.4 APPLICATION OF THE RESULTS OF A SPLICE ANALYSIS

Example

The effective areas of the splice of Table III.2 would be determined in a manner similar to that used for the doubler. The calculations are shown in Table VI.2. The effective area (and width) of both splice members (S and D) are determined. These would then, in any future analyses of the whole structure, be considered as one integral number.

TABLE VI.2

DETERMINATION OF THE EFFECTIVE AREA AND EFFECTIVE WIDTH OF A SPLICE

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
STA.	$A_D E_D$	E_D	t_D	$A_S E_S$	E_S	t_S	$A_D E_D + A_S E_S$	APPLIED	P_{O_D}	P_D	EFF. $A_D E_D$ *	EFF. A_D	EFF. WIDTH OF D	P_{O_S}	P_S	EFF. $A_S E_S$ **	EFF. A_S	EFF. WIDTH OF S
	TABLE III.2	DATA	DATA	TABLE III.2	DATA	DATA	2+5	TABLE III.2	$\frac{2}{8}\times 9$	TABLE III.2	$\frac{11}{10}\times 2$ *	$\frac{12}{3}$	$\frac{13}{4}$	$\frac{5}{8}\times 9$	TABLE III.2	$\frac{16}{15}\times 5$	$\frac{17}{6}$	$\frac{18}{7}$
	$x10^{-6}$	$x10^{-6}$		$x10^{-6}$	$x10^{-6}$		$x10^{-6}$				$x10^{-6}$					$x10^{-6}$		
1	4.7	10	.10	4.7	10	.10	9.4	8,000	4,000	416	.49	.049	.49	4,000	7,584	4.7**	.470	4.70
2	"	"	"	"	"	"	"	"	"	2.012	2.35	.236	2.36	"	5,988	4.7**	.470	4.70
3	"	"	"	"	"	"	"	"	"	2,932	3.44	.344	3.44	"	5,068	4.7**	.470	4.70
4	"	"	"	"	"	"	"	"	"	3,488	4.10	.410	4.10	"	4,512	4.7**	.470	4.70
5	"	"	"	"	"	"	"	"	"	3,869	4.44	.444	4.44	"	4,131	4.7**	.470	4.70
6	"	"	"	"	"	"	"	"	"	4,205	4.7*	.470	4.70	"	3,794	4.45	.445	4.45
7	"	"	"	"	"	"	"	"	"	4,610	4.7*	.470	4.70	"	3,390	3.99	.399	3.99
8	"	"	"	"	"	"	"	"	"	5,220	4.7*	.470	4.70	"	2,780	3.25	.325	3.25
9	"	"	"	"	"	"	"	"	"	6,245	4.7*	.470	4.70	"	1,755	2.06	.206	2.06

* If Column 12_n is greater than Column 2_n, use Column 2_n value for Column 12_n.

** If Column 17_n is greater than Column 5_n, use Column 5_n value for Column 17_n.

The total effective area of the splice at any station, n, is the sum, $13_n + 18_n$.

VI.5 ECCENTRIC DOUBLER INSTALLATIONS

Another type of problem involving the effective area of a doubler would occur when an external doubler is attached over a stringer-skin element. In this case the eccentricity of the (single) doubler would affect the stress level and it could result in significant bending stresses being present due to the installation. Such stresses could be quite important if either fatigue life or compressive strength were the reason for adding the doubler. That is, in the fatigue case the bending stresses due to the eccentricity might need to be accounted for, and in the compressive strength case the beam-column effect due to the eccentricity should always be considered.

For common engineering purposes, a method of accounting for the effect of the single (or "eccentric") doubler would be as follows:

a. As discussed previously, (Table VI.1) determine the effective area distribution of the doubler and consider this to be integral with the base structures (the stringer-skin element).

b. Determine the centroid distribution of this integral unit. (This centroid will not coincide with that of the original skin stringer element.) These centroids establish the neutral axis of the integral unit.

c. Carry out a conventional analysis of the effective structure which now has a "bent shape" for the neutral axis of the integral unit (members attached to the doubler). In this analysis

   (1) There will be an "initial" bending moment, $P \cdot e_x$, where P is the axial load and e_x is the distance between the centroid line and the load line at any station x. (The centroid line is obtained by considering only the effective area of the doubler together with the actual base structure.)

   (2) The moment of inertia of the cross section, however, will include all of the doubler cross section (not just the effective area, which is used only in determining e_x in (1) above). That is, the usual engineering bending theory is assumed to apply for the calculations involving bending.

   (3) The actual analysis (a beam-column analysis, or a beam in tension analysis) will then be an iterative

procedure* beginning with the applied axial load P and the initial bending moments, at any station, x, given by

$$M_x = P \cdot e_x$$

As in all such analyses, it is necessary to consider some of the structure beyond the members attached to the doubler, but this depends upon the analyst's judgment and the degree of accuracy required. The results give the final bending moments, M', along the members, enabling the total stresses

$$f = \frac{P}{A} \pm \frac{M'c}{I}$$

to be calculated. The fatigue life, the yield strength or the ultimate strength can then be assessed.

VI.6 ECCENTRIC (SINGLE LAP) SPLICE INSTALLATIONS

The remarks of Article VI.5 above would also apply to a single lap splice installation.

* Since the effective members are tapered, EI is not constant and hence the standard formulas for beam-columns (with either compressive or tensile axial loads) do not apply. Hence, either "average" constant EI values must be assumed for solution by formulas, or else an iterative (numerical) procedure must be used to determine the final bending moments. A practical engineering method for such numerical beam-column analyses is presented and illustrated in Reference (10).

SECTION VII

TEST PROGRAM

VII.1 INTRODUCTION

In order to accomplish the purposes of this report, the test program described below was conducted. Since there is such a large number of suitable types and sizes of fasteners, sheet gages, hole clearances, etc., the test program was generally limited to one representative fastener for the various assembly tests. The protruding head Hi-Lok Pin was used since it is a widely used, stiff and permanent type. The tests and test specimens are of two general types, assembly tests and element tests. The assembly tests were conducted to verify the methods of analyses. The element tests were conducted to obtain specific data necessary for the predictive analyses of the assemblies tested.

VII.2 ASSEMBLY TESTS AND SPECIMENS

The purpose of the Assembly Tests was to verify experimentally the methods of analysis. In these tests doubler and splice assemblies were loaded in a tension test machine and the distributions of internal loads were obtained by using photostress plastic and methods. There were two types of Assembly Tests.

a. Doubler Assembly Tests

b. Splice Assembly Tests

Fifteen assembly tests were made using specimens having 5/32" diameter Hi-Lok (HL1870) Fasteners of the protruding head type. Three tests involved specimens having 1/4" bolts and two tests were made using spotwelded doubler assemblies. 7075-T6 Al. alloy sheet material was used in all Assembly Test Specimens.

VII.3 DOUBLER ASSEMBLY SPECIMENS

Details of these are shown in Figures VII.1 through VII.4. There are 13 specimens. Except where noted otherwise, the fasteners were 5/32" Hi-Lok 1870 and the holes were reamed for a sliding fit (no "slop"). Photostress plastic was applied to the outer surface of each member of single lap specimens and to the outer surface of one of the outside members of all sandwich specimens except when it was applied to the outer surface of both outside members.

a. Specimen I-A1

(1) This specimen is as sketched in Figure VII.1 except that there were only 10 fasteners, spaced at 2 inches.

(2) The purpose was to verify the methods of analysis using a uniform specimen and a wide fastener spacing.

b. Specimen I-A2

(1) This specimen was as sketched in Figure VII.1.

(2) The purpose was the same as for I-A1, using a closer fastener spacing.

c. Specimen I-B1

(1) This specimen was identical to I-A2 except that there were two doublers (a "sandwich").

(2) The purpose was

(a) The same as I-A1 and
(b) To reduce the effects of eccentricity.

d. Specimen I-B2

(1) This specimen was the same one as I-B1 except that the second and third fastener holes at one end only were reamed 0.005" oversize for this test.

(2) The purpose was

(a) To illustrate the effect of hole clearance ("slop") and the method of accounting for it.
(b) To verify the method of analysis using an unsymmetrical specimen.

e. Specimen I-C

(1) This specimen was as sketched in Figure VII.2.

(2) The purpose was to verify the method for a tapered member and for a specimen having multi-fastener rows.

f. Specimen I-D1

(1) This specimen was identical to I-C except that there were two doublers (a sandwich).

(2) The purpose was to reduce the effects of eccentricity.

g. Specimen I-D2 (I-D1 re-used)

(1) This was the same as specimen I-D1 except that the 7th and 9th rows of fasteners (from both ends) were not installed.

(2) The purpose was to illustrate that fewer (and, hence, smaller) fasteners can be used near the center with little effect on internal loads.

h. Specimen I-E

(1) This specimen was as sketched in Figure VII.3.

(2) The purpose was to show the effect of a "wide" base structure, to verify the method of analysis, and to define the fastener load diffusion rate into the base structure.

i. Specimen I-F

(1) This specimen is as sketched in Figure VII.4, a "stacked" doubler.

(2) The purpose is to evaluate the suggested method of analyzing such cases.

j. Specimen I-G1

(1) This specimen is identical to I-A2 except that spotwelds are used instead of HL 1870 Rivets.

(2) The purpose is to verify the applicability of the analyses to spotwelded assemblies.

k. Specimen I-G2

(1) This specimen is identical to I-B1 except that spotwelds are used instead of HL 1870 Rivets.

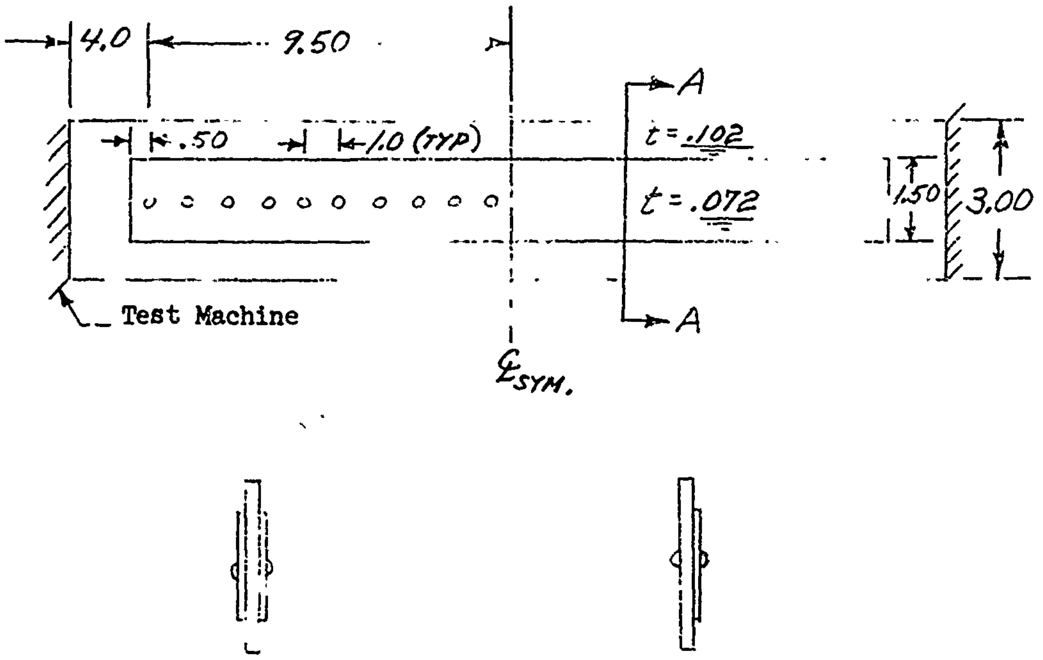
(2) The purpose is to reduce the effects of eccentricity.

l. Specimen I-H1

This specimen is similar in design and purpose to Specimen I-B1, but 1/4" NAS Bolts and AN 320 Nuts (fingertight) were used instead of the HL 1870 Rivets.

m. Specimen I-H2

This specimen is similar in design and purpose to Specimen I-B2, but 1/4" NAS Bolts and AN 320 Nuts (fingertight) were used instead of the HL 1870 Rivets.

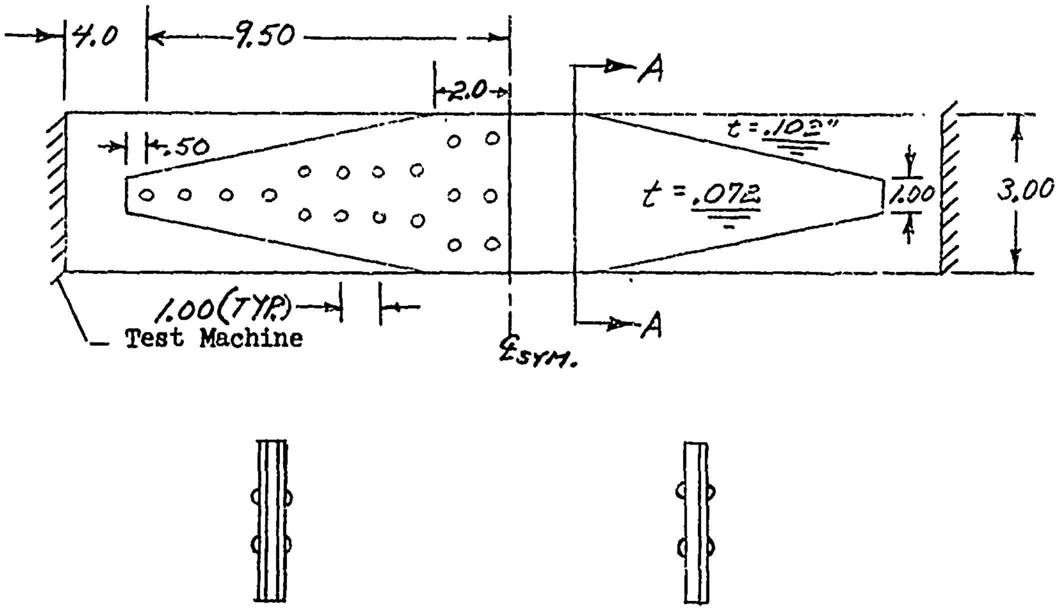


Specimens B1,B2,G2,H1 & H2

Specimen A1,A2,G1

View A-A

Figure VII.1. Constant Width Doubler Specimens



Specimens D1 and D2

Specimen C

View A-A

Figure VII.2. Tapered Planform Doubler Specimens

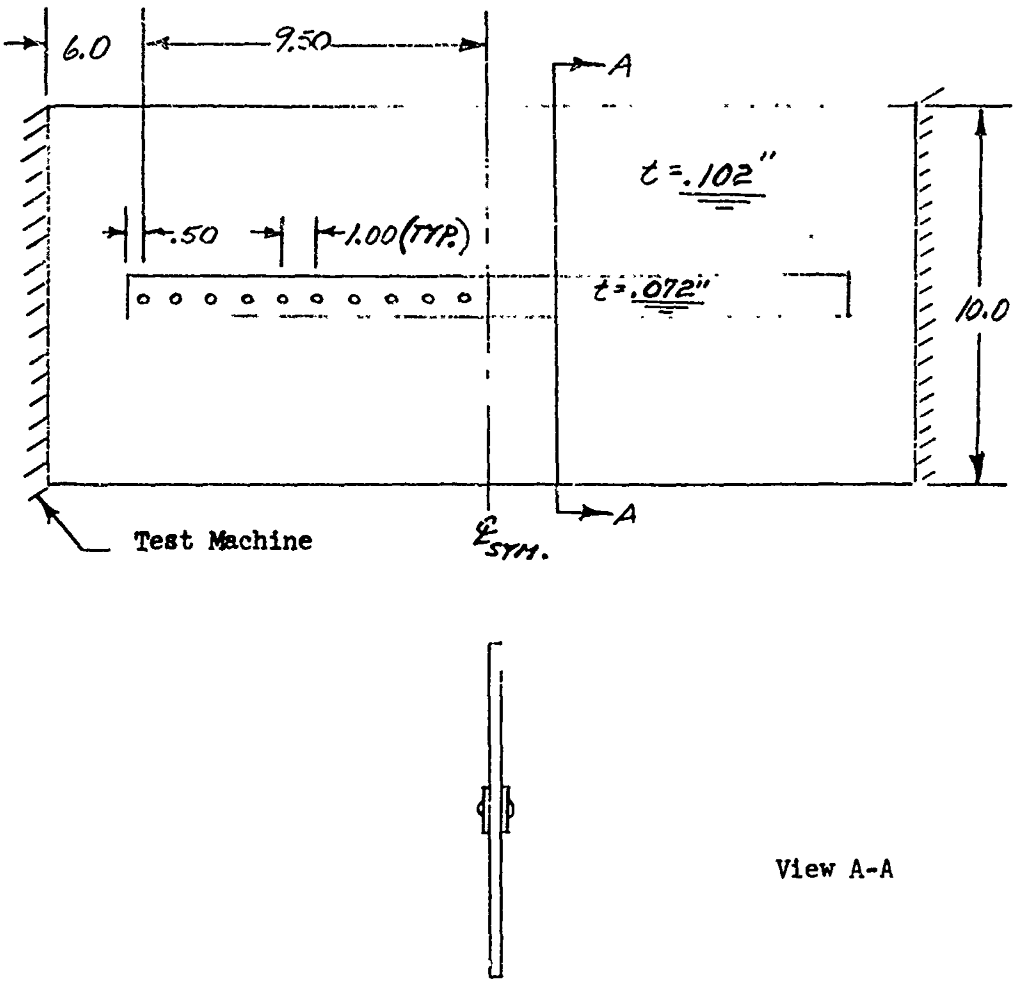


Figure VII.3 Wide Base Structure Specimen I-E

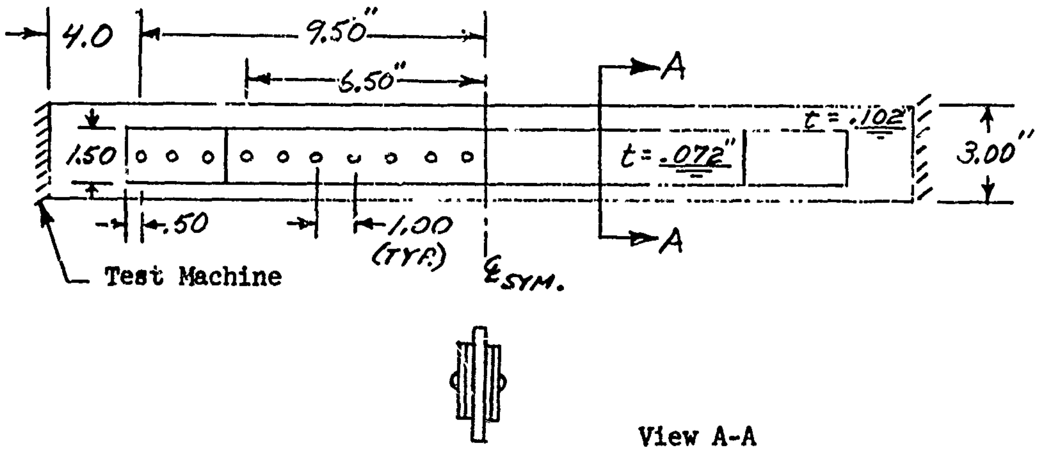


Figure VII.4 "Stacked" Doubler Specimen I-F

VII.4 SPLICE ASSEMBLY TEST SPECIMENS

Details of these are shown in Figures VII.5 --- VII.7. There are seven specimens. Except where noted otherwise the fasteners were 5/32" HL 1870 and the holes were reamed for a sliding fit (no "slop"). Photostress plastic was applied in the same manner as for the doubler assembly specimens.

a. Specimen II-A1

(1) This specimen is as sketched in Figure VII.5 except that there are six fasteners at a 2 inch spacing.

(2) The purpose is to verify the methods of analysis.

b. Specimen II-A2

This specimen is the same one as for II-A1 except that there are 12 fasteners at a 1" spacing.

c. Specimen II-B1

(1) This specimen is as illustrated in Figure VII.5, a sandwich.

(2) The purpose is to reduce the eccentricities present in II-A2.

d. Specimen II-B2

(1) This specimen is the same as II-B1 except that the second and third fastener holes at one end only were reamed 0.005" oversize.

(2) The purpose is to illustrate the effect of fastener-hole clearance and also an unsymmetrical case.

e. Specimen II-C1

(1) This specimen is as illustrated in Figure VII.6.

(2) The purpose is to verify the method for a tapered member and also for a case involving multi-fastener rows.

f. Specimen II-C2

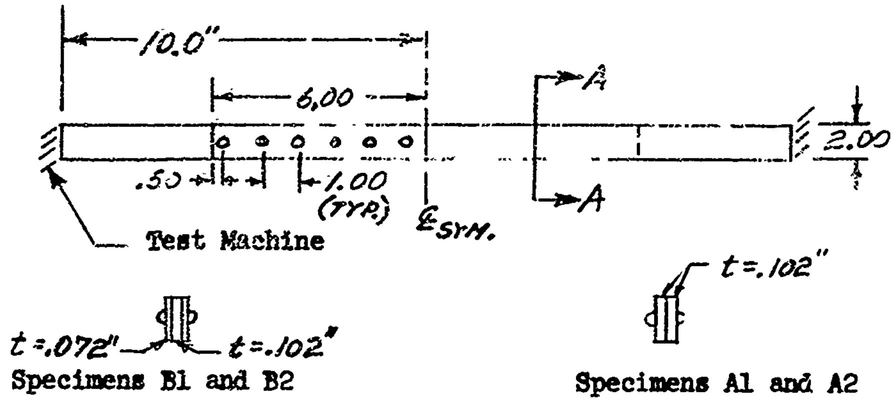
(1) This specimen is identical to II-C1 except that it is a sandwich.

(2) The purpose is to reduce the eccentricities present in II-C1.

g. Specimen II-D

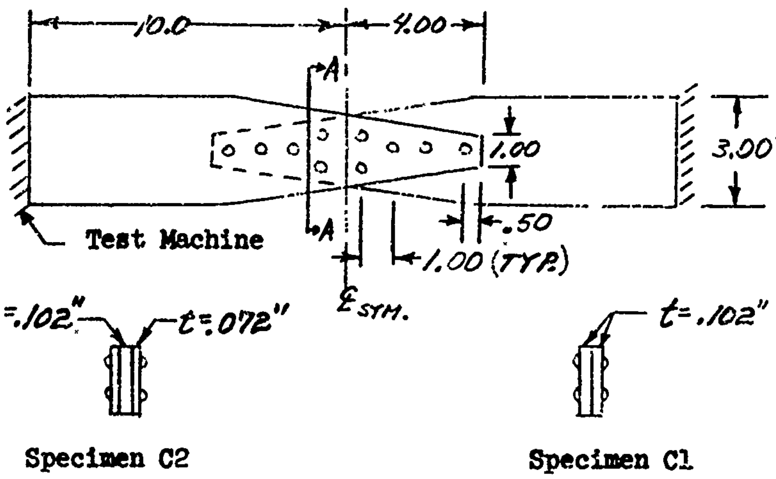
(1) This specimen is as illustrated in Figure VII.7. The AN 320 Nuts are installed fingertight.

(2) The purpose is to illustrate a "short splice" without clamping friction.



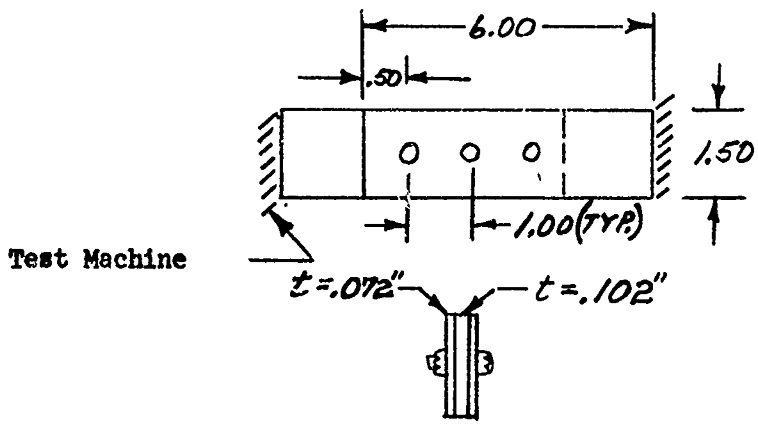
View A-A

FIGURE VII.5. Constant Width Splice Specimens



View A-A

Figure VII.6 Tapered Planform Splice Specimens



View A-A

Figure VII.7. Short Bolted Splice Specimen II-D

VII.5 INDIVIDUAL (ELEMENT) TEST SPECIMENS

In order to obtain the specific data necessary for predicting the internal loads in the various test assemblies, the following element tests were required. Most of these were for the purpose of obtaining the load-deflection curves (fastener spring constants) for the selected sheet thickness and fastener hole sizes. These tests were made using the same type of specimen (and test) that is conventionally used at Vought Aeronautics Division to obtain fastener-sheet load-deflection data. It has been found previously that three specimens of any fastener-sheet combination must be tested to obtain sufficient data to define the relationship accurately. The specimens of this type are referred to as Type III and are described below. All sheet material was 7075-T6 aluminum alloy. All HL 1870 Fasteners are 5/32" diameter.

a. Specimen III-A1

   One HL 1870 Rivet fastening two 0.072" sheets, hole reamed for sliding fit.

b. Specimen III-A2

   One HL 1870 Rivet fastening two 0.102" sheets.

c. Specimen III-A3

   One HL 1870 Rivet fastening a 0.102" and a 0.072" sheet.

d. Specimen III-A4

   One HL 1870 Rivet fastening a sandwich of two 0.072 sheets and one 0.102 sheet.

e. Specimen III-B1 through III-B4

   Same as III-A1 through III-A4 but holes reamed for 0.005" clearance.

f. Specimens III-C1 through III-C4

   Same as III-A1 through III-A4 but using NAS 464 and AN 364 Shear type Nuts (and washer) with nut fingertight. (1/4" Bolts).

g. Specimens III-D1 through III-D4

   Same as III-C1 through III-C4 but with nuts torqued to 35 in/lbs.

h. Specimen III-A5

One HL 1870 Rivet fastening a double sandwich of four 0.072" sheets and one 0.102 center sheet. The center sheet is not loaded.

i. Specimen III-E1 through III-E4

Same as III-C1 through III-C4 but with holes reamed for 0.005" clearance.

j. Specimens III-fL through III-F4

Same as III-E1 through III-E4 but wuth nuts torqued to 35 in/lbs.

k. Specimen III-G

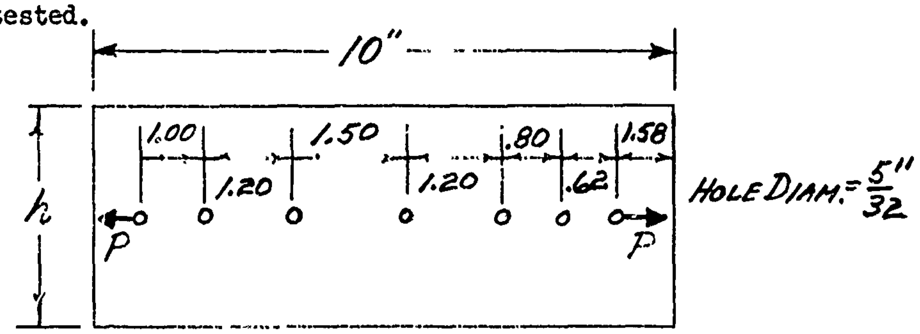
Same as III-A1 but using spotwelds instead of HL 1870 Rivets.

l. Specimen III-H

Same as III-A4 but using spotwelds instead of HL 1870 Rivets.

VII.6 PHOTOSTRESS PLASTIC TEST SPECIMENS

These tests were made using photostress material, as shown in Figure VII.8. The three photostress plastic specimens shown in Figure VII.8 were tested.



SPECIMEN	h
A	10"
B	7"
C	3"

Figure VII.8 Photostress Plastic Test Specimens

The purpose of these tests was to help define

a. Rate at which the fastener load "diffuses" into the sheet.

b. The "dead" area between the holes (as a percent of the fastener diameter).

VII.7 TESTING PROCEDURES

a. Load-Deflection Tests

Each of the specimens of Type III was mounted in a suitable tension testing machine and load-deflection data was obtained using an autographic recorder. (Figures VII.9 and VII.10 show typical results.)

b. Doubler and Splice Assembly Specimen Tests

Each of the specimens of Types I and II was mounted in a suitable tension testing machine and loaded successively to the three values specified in Table VII.1. Each load was released before proceeding to the subsequent one. Color photographs of the photostress plastic strain distribution were obtained for each loaded and unloaded condition.

TABLE VII.1

TEST LOADS FOR ASSEMBLY SPECIMENS

SPECIMEN	APPLIED TEST LOAD			SPECIMEN	APPLIED TEST LOAD		
	Q1	Q2	Q3		Q1	Q2	Q3
I-A1	7,120	10,910	18,000	II-A1	3,330	5,620	11,910
I-A2	9.210	14,150	18,000	II-A2	4,800	8,150	12,000
I-B1	6,760	12,300	18,000	II-B1	4,530	8,224	12,000
I-B2	5,670	11,240	18,000	II-B2	3,790	7,525	12,000
I-C	8,660	13,290	18,000	II-C1	5,920	9,320	18,000
I-D1	6,540	11,870	18,000	II-C2	5,555	10,039	18,000
I-D2	6,520	11,890	18,000	II-D	2,655	6,021	--
I-E	18,950	34,517	60,000				
I-F	12,400	18,000	--				
I-G1	3,802	7,000	13,550				
I-G2	3,640	7,330	15,890				
I-H1	6,280	14,520	18,000				
I-H2	5,190	13,490	18,000				

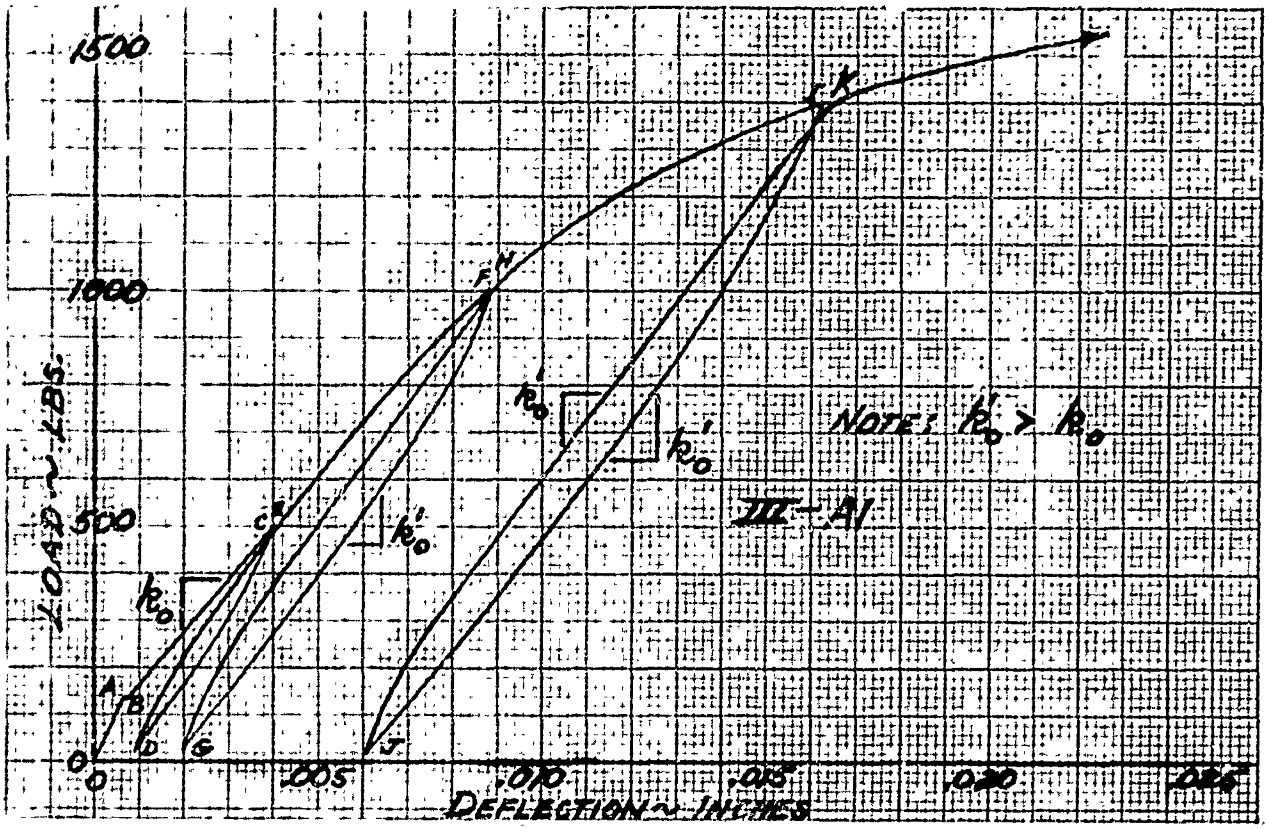


Figure VII.9. Load-Deflection Plots From the Autographic Recorder

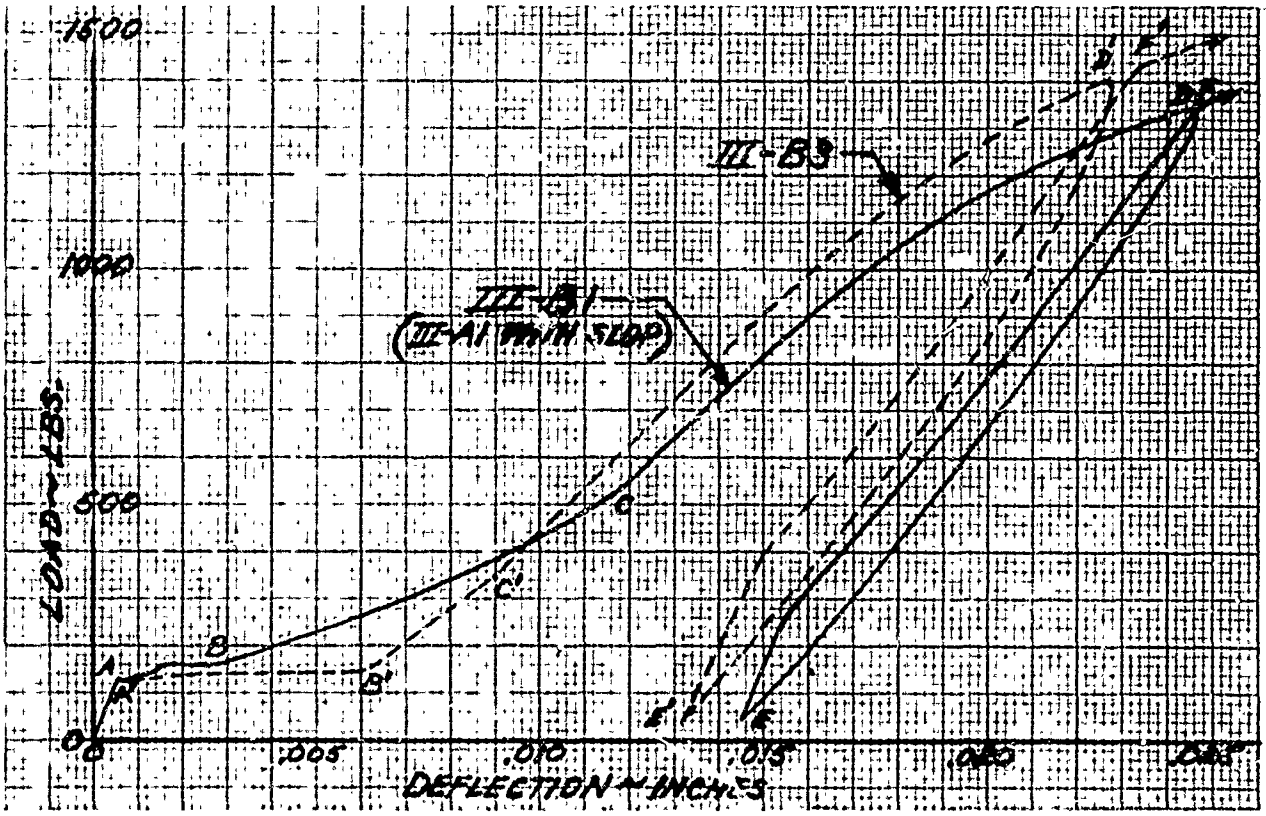


Figure VII.10. Load-Deflection Plots From the Autographic Recorder

Then, using photostress analysis methods, the internal loads at selected stations were determined for all specimens. The results are presented in Table VII.2 together with the "predicted" loads for the purposes of comparison. Pictures of some typical photostress plastic strain distributions are shown in Figures VII.19, VII.20, and VII.21.

c. The photostress plastic specimens of Figure VII.8 were tested as follows:

(1) Each specimen having only the end holes drilled was mounted in a loading apparatus. A tensile load, P, was then applied of sufficient magnitude to obtain a well-defined color photograph of the resulting strain distribution in the specimen.

(2) Step (1) was repeated for a compressive load, -P.

(3) Step (1) was repeated after drilling the additional holes in the specimen.

(4) Step (3) was repeated for a compressive load, P.

(5) Equal tensile loads, P, were then applied at the two holes at each end (4 loads, P) and a color photograph of the resulting strain distribution was obtained.

A typical photograph is shown in Figure VII.18.

VII.8 TEST RESULTS

a. Load-Deflection Tests

Some typical load-deflection curves, as obtained directly from the autographic recorders, are presented in Figures VII.9 and VII.10. Although all tests were carried to failure, the deflections at these points were beyond the limits of the recorder. In Figure VII.9 OA shows the initial stiffness due to friction, AB shows a slight slip when friction is overcome, and BC shows the steady linear rise to C where the applied load is reduced. The specimen then unloads at a faster rate, CD, than it loaded up, BC. (An initial loading of about 50 pounds is held on the test machine.) Then, as the loading is increased, DE shows the action in "returning" to the basic curve of which EF is a continuation. Similar action continues from point F on until $P_{Max.}$ is obtained. Thus, it is

seen that the "loops" CED, FGH, and IJK represent a hysteresis effect always present, even at low load levels in the initial linear range. The average slope of the linear portion (the "sides") of these loops is referred to as the secondary spring constant, k'_o, and this is seen to be larger than the initial (linear) spring constant, k_o. Actually, k'_o is largest when obtained well out in the plastic range, but most of the increase ($k'_o - k_o$) is obtained early in the region of the initially linear portion of the load-deflective curve. The values of k'_o reported are obtained from "loops" that are somewhat past the "knee" of the load-deflection curve. As can be seen from Figure VII.10 (and also in later figures), k'_o is only slightly affected (reduced) by slop. Although k'_o may be as much as 50% larger than k_o for certain combinations, this value is not usually presented in reporting fastener-sheet load-deflection results. However, using k_o in determining residual loads does not, fortunately, result in significantly large errors and this usage is suggested when k'_o is unknown.

The solid curve of Figure VII.10 shows what happens when a specimen, III-A1, is manufactured with a slop of approximately 0.005 inches. There is the initial friction OA, the slipping AB, and a transition, BC, to the basic curve CD. From C on the action is similar to that of a specimen having no initial slop. The dashed curve is for a different specimen. Here the slipping A'B' is more as would be expected (about 0.005"). This is followed by a steeper transition, B'C', to the basic curve CD. Actually the two curves shown represent the extremes in the region ABC for specimens having 0.005" initial slop.

Figures VII.11 through VII.17 present the "final" load-deflection curves for the various types of joints tested. Each of these has been obtained as follows:

(1) The outer envelope, KIHFECA, as in Figure VII.9, was "smoothed out" for three similar specimens tested. The portion CA was extrapolated to intersect the abscissa (at a point to the left of zero), thereby eliminating the friction effect. This extrapolation established a new origin for the curve.

(2) The results of this procedure for the three specimens were averaged to obtain the "final" load-deflection curve for the joint.

This procedure can be seen by comparing the "final" curve for specimen III-A, (Figure VII.11) with one of the test curves for III-A, (Figure VII.9).

For the cases of specimens having slop, the same procedure was used except that, as in Figure VII.10, the portion DC or D'C' was extrapolated to intersect the abscissa (to the right of zero). This procedure thus establishes a new origin and removes the "slop". (The slop is then considered separately as discussed in Section III.) The results of this procedure can be seen by comparing the test results for specimen III-B1 and III-B3 (Figure VII.10) with the "final" load-deflection curves presented in Figure VII.12. The "final" curves of Figure VII.12 are thus for such joints after the applied loads are large enough to "close up" any initial slop in the actual structural assembly, and they are, specifically, for the 0.005" initial slop in these tests.

An alternate method of considering the slop effect would be, in Figure VII.10, to simply draw a straight line from O to C, or to C'. This would result in a load-deflection curve having an unchanged origin, OCD etc., but it could not be used with the simpler analysis of Articles III.2 and III.3 (for the elastic range). That is, the superposition approach of Article III.6 would always be required because of this initial small slope of the curve. Actually, in practice, there will seldom, if ever, be available any specific load-deflection curves of this type. That is, only the load-deflection curves for "tight" joints can be expected, and even these are not at present generally available for many fasteners. Hence, in most cases, the analyst must use these curves and consider the slop as discussed in Section III.

The "final" load-deflection curves derived from the load-deflection tests are presented in Figures VII.11 through VII.17. Each of these curves has been obtained by averaging the load-deflection data from the tests of three similar specimens. An inspection of these results shows how some of the parameters such as sheet thickness, single and double lap, fastener size (1/4" bolts and 5/32" rivets) clamp-up (bolt torque-up) and "slop" affect the stiffness of the joint as discussed in Section V. In the case of fasteners with slop, the slop has been removed from the results as discussed previously. The maximum load for each specimen is also indicated. However, this occurs at a large deflection (as does the maximum stress in a typical ductile material stress-strain curve) that is beyond the limits of the test machine plotting equipment. For

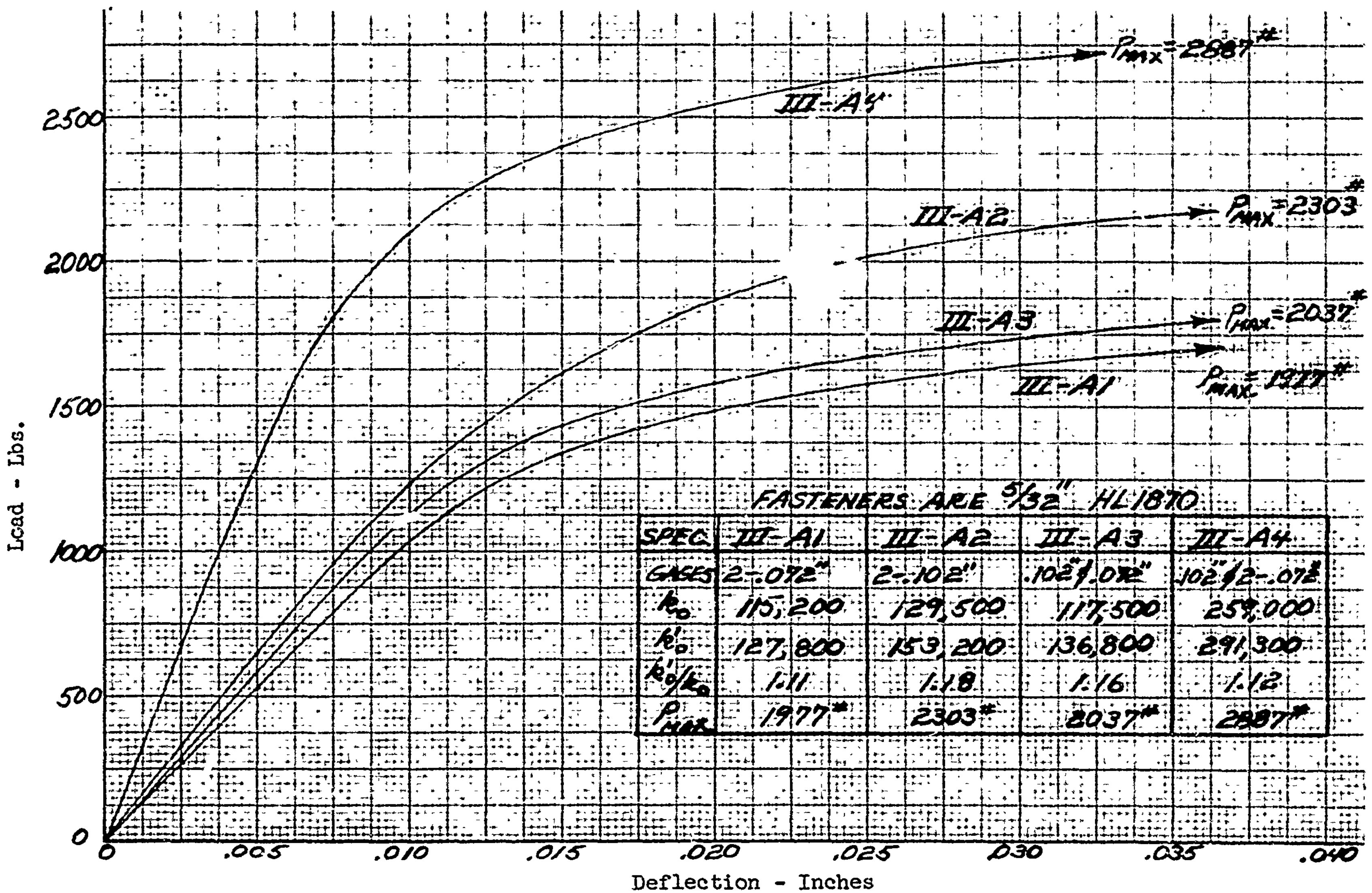
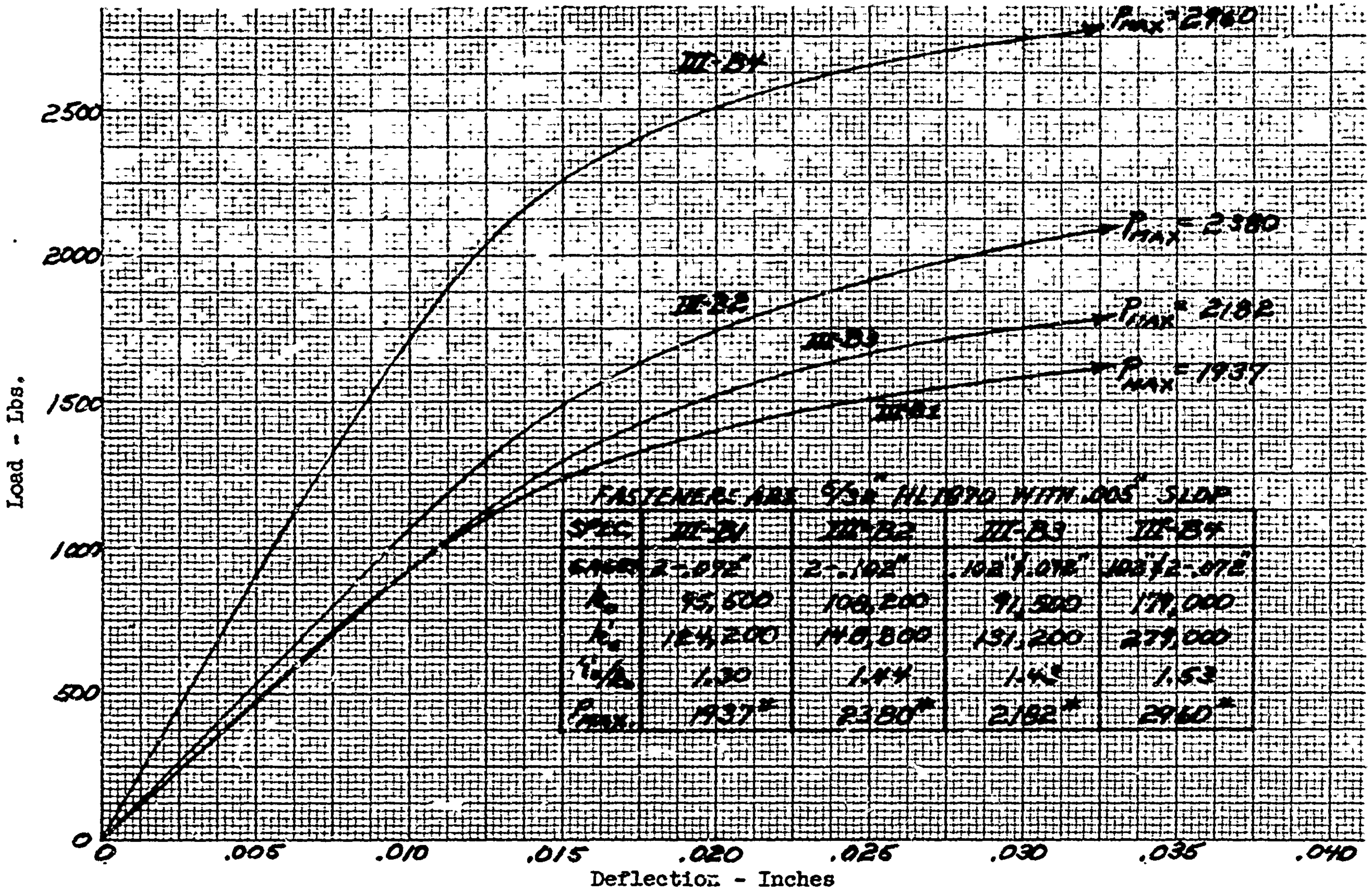


Figure VII.11 Load-Deflection Curves -- HL1870 Fasteners Having Sliding Fit



SPEC	III-B1	III-B2	III-B3	III-B4
GAGE	2-.072"	2-.102"	.102"/.072"	.102"/2-.072"
k_o	95,600	108,200	91,500	177,000
k_o'	124,200	148,800	131,200	271,000
k_o'/k_o	1.30	1.44	1.43	1.53
P_{MAX}	1937#	2380#	2182#	2960#

Figure VII.12 Load-Deflection Curves -- HL1870 Fasteners Having 0.005" Initial Slop

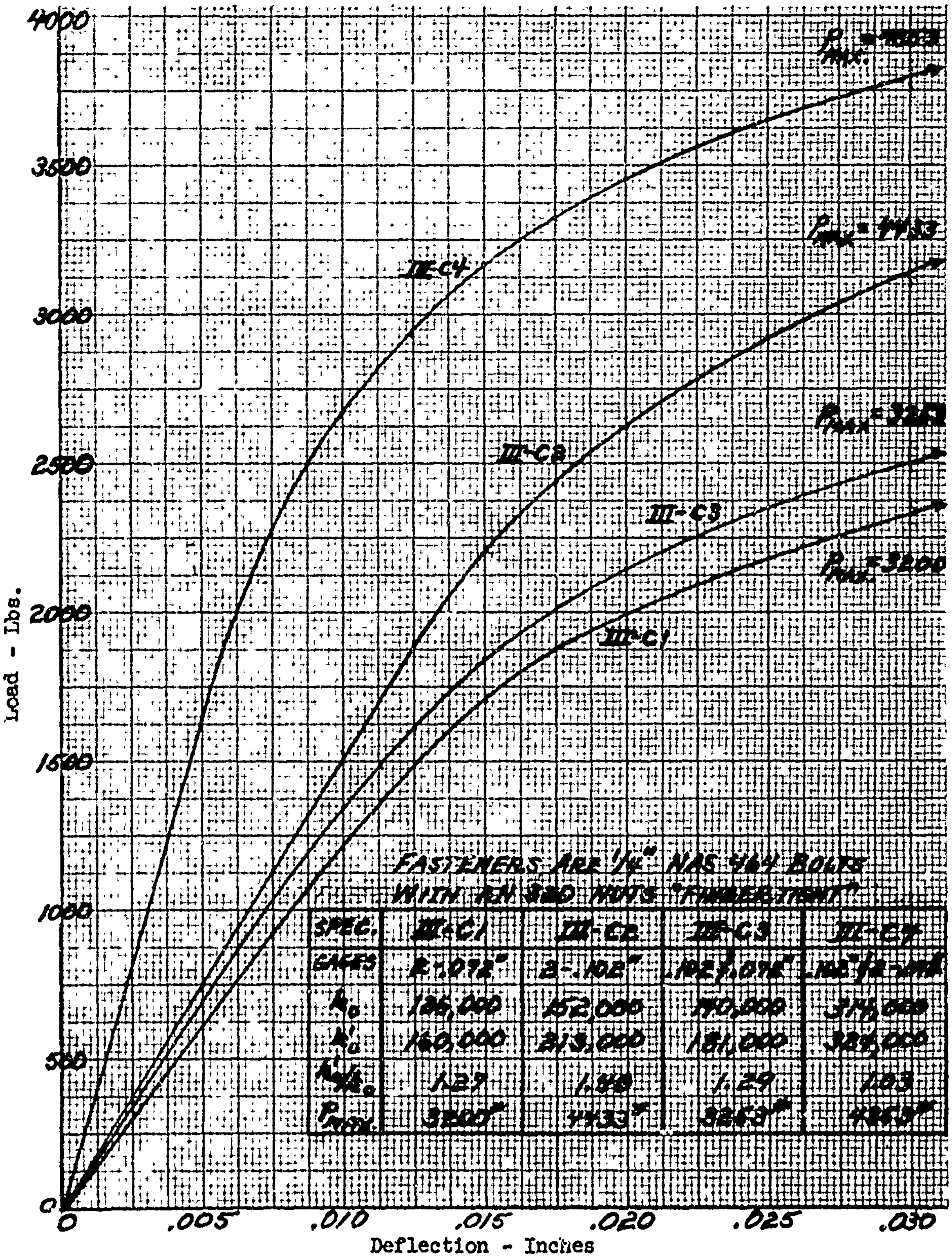
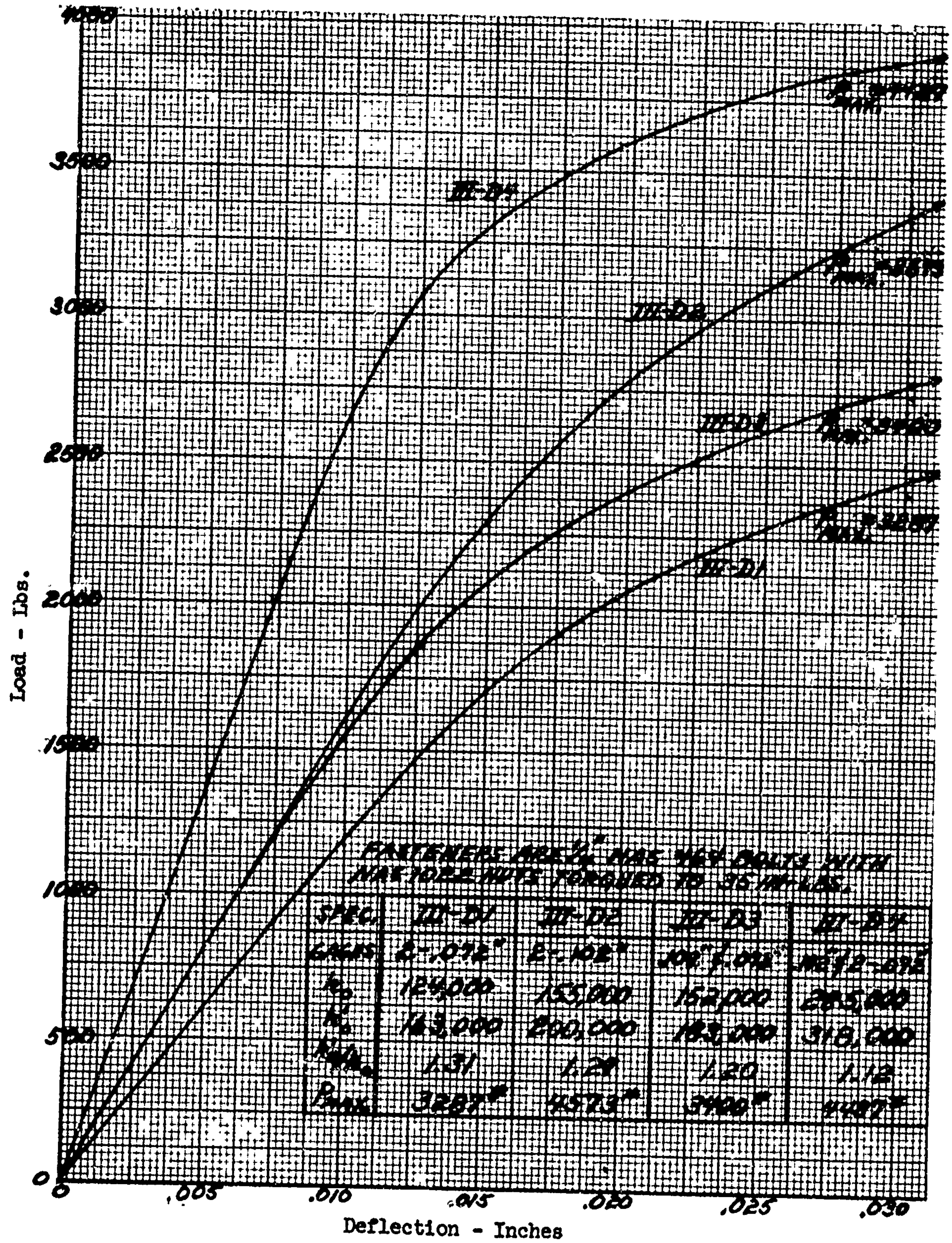


Figure VII.13 NAS Bolts Having a Sliding Fit and Fingertight Nuts



SPEC.	III-D1	III-D2	III-D3	III-D4
GAGES	2-.072"	2-.102"	.102"/.072"	.102"/2-.072"
k_o	124,000	155,000	152,000	285,000
k_o	163,000	200,000	183,000	318,000
k/k_o	1.31	1.29	1.20	1.12
P_{MAX}	3287#	4573#	3400#	4427#

Figure VII.14 Load-Deflection Curves -- NAS Bolts Having a Sliding Fit and Torqued Nuts

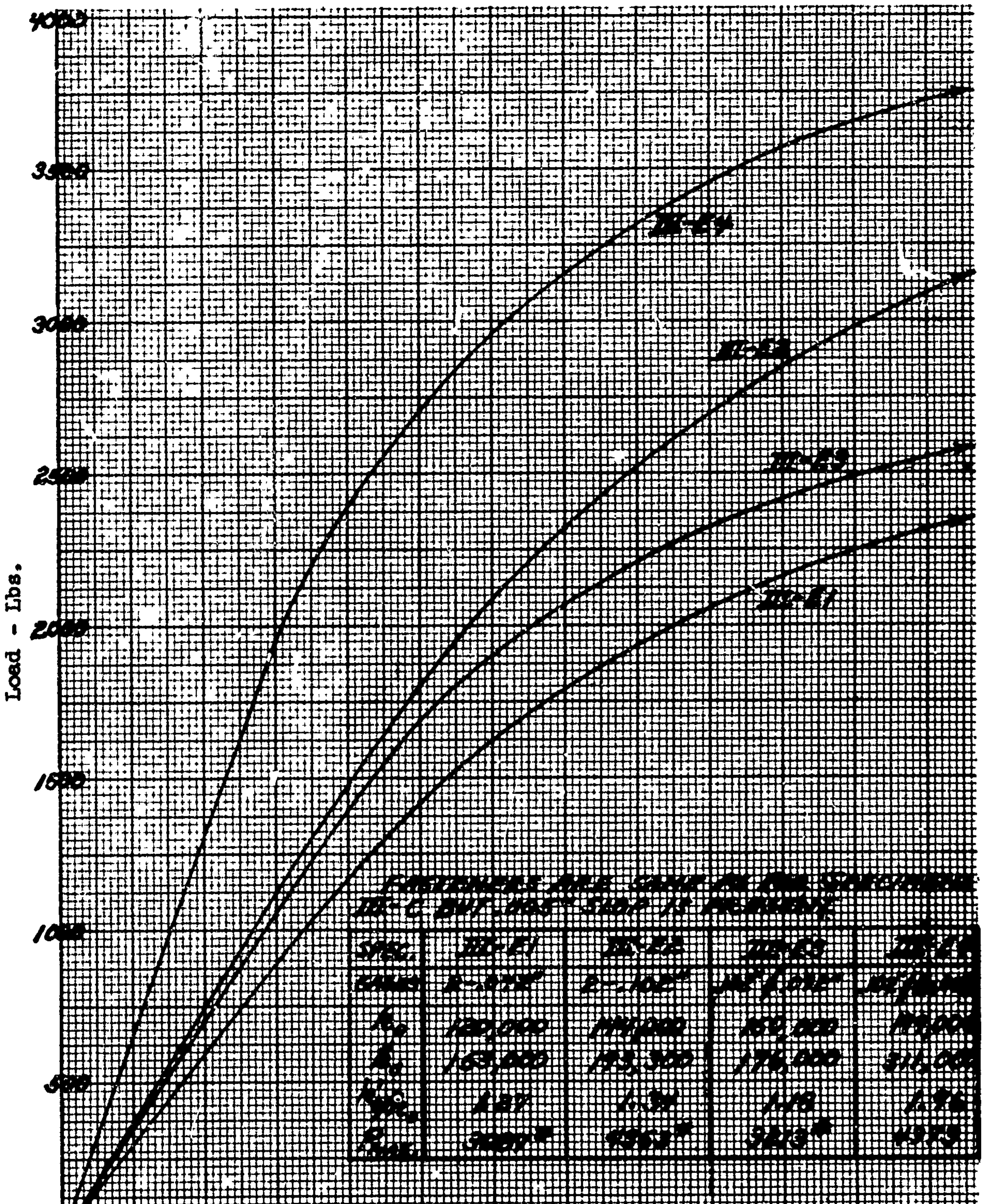


Figure VII.15 Load-Deflection Curves -- NAS Bolts Having 0.005" Initial Slop and Fingertight Nuts

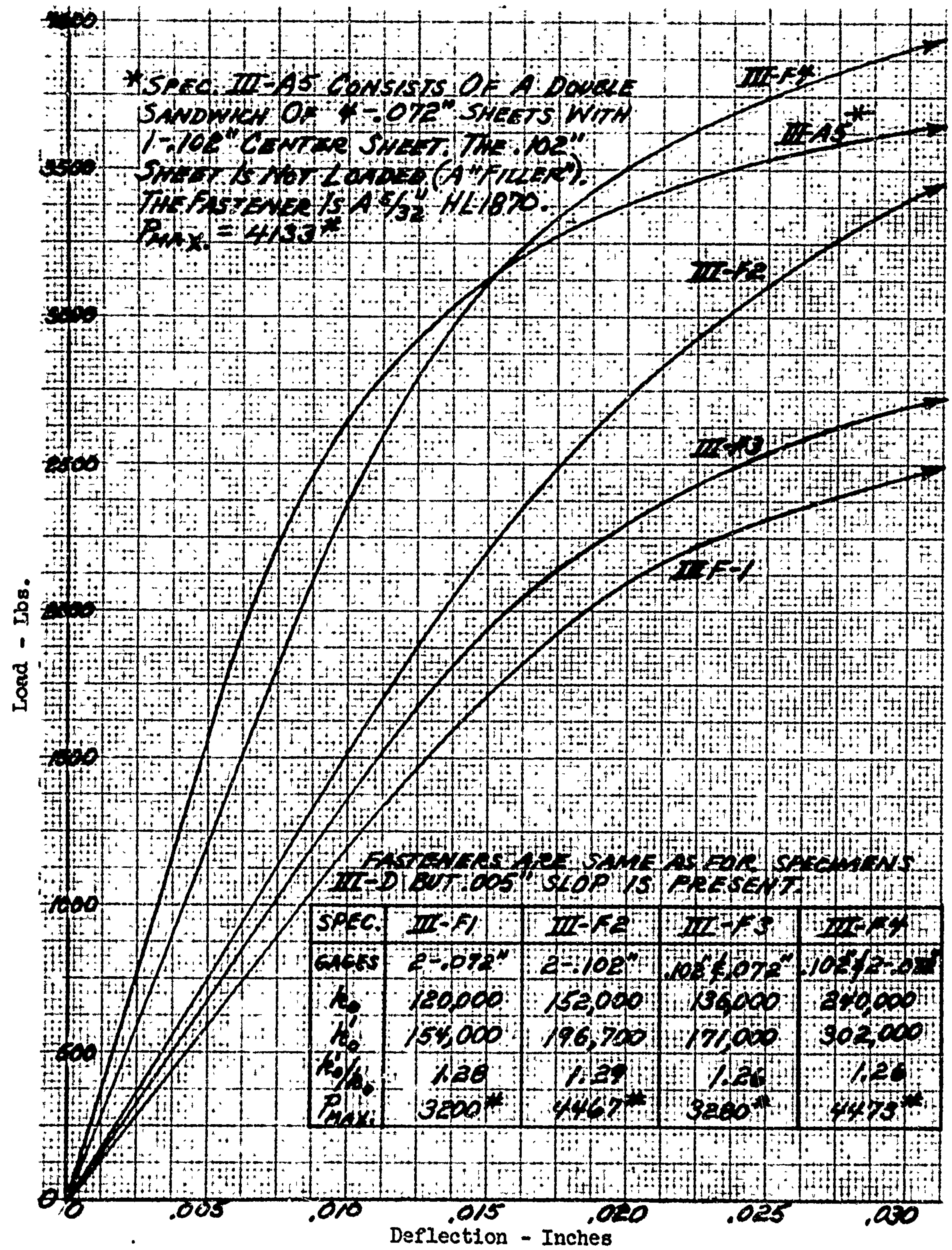


Figure VII.16 Load-Deflection Curves -- NAS Bolts Having 0.005" Initial Slop and Torqued Nuts

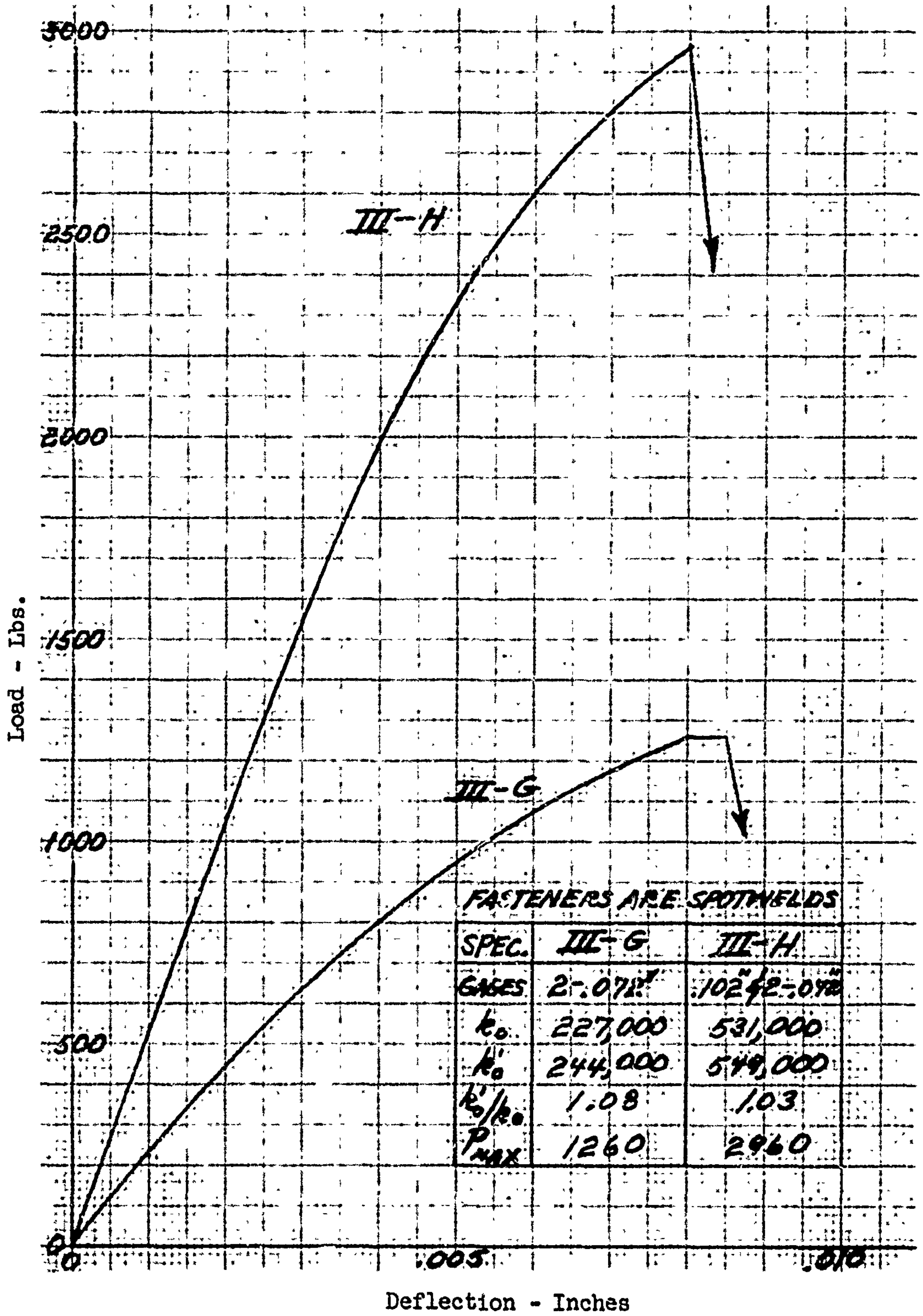


Figure VII.17 Load-Deflection Curves -- Spotwelds

example, P_{max}, for the specimens of type III-A (Figure VII.11) is estimated to have occurred at a deflection of about 0.10" - 0.15", or at 3 to 4 times the deflection range shown on the graph.

Not all of the fastener combinations tested were used in the assembly test specimens (Types I and II) but have been included in the test program to show the effects of the various parameters. The results for specimens II-A1-A3, III-C1-C3 and III-D1-D3 indicate the reasonableness of obtaining k_o for a joint of two different thicknesses as suggested in Article V.2. They also show that the "secondary" spring constant, k'_o, can be estimated in this manner.

The results for the spotwelded sheet combinations, Figure VII.17, show the joint to be of a brittle nature as would be expected. There is no significant plasticity as in the more "ductile" mechanically fastened joint. (However, if the mechanical joint is critical in shear rather than bearing, it becomes "brittle" like the spotweld.) Although the actual spotweld load-deflection curve was used for predicting the internal loads, it would probably be sufficient to simply replace it with a straight line having the initial slope and the maximum value of P_{max} shown.

b. Doubler Assembly Tests

The results of these tests are presented in Table VII.2. For purposes of comparison both the test loads and the predicted loads are tabulated. The three (or four in some cases) outer fastener loads at one end and the maximum load in the doubler are listed. The fastener loads were obtained as the difference between the loads in the doubler at successive stations midway between the fasteners. The doubler loads at these stations are not listed but were obtained at each station by

(1) determining the stress at five points across the member by means of a photostress analysis. This was actually done making a visual point analysis while the specimen was strained in the test machine. However, the analysis can also be made from the color photographs obtained.

(2) plotting these stress levels to establish a curve showing the stress variation across the member

(3) Integrating this curve to obtain the total load in the member at the selected station. This load is, therefore, based upon the stress in the outer surface of the member and includes any bending stresses present. It does not separate the

bending stresses.* For illustrative purposes the predicted residual loads are also listed. These are small except where significant yielding has occurred at the larger applied loads. The test values of the residual loads, where significant, were also estimated from the color photographs.

In order to demonstrate the effect of using k'_{F_O}, the secondary fastener spring constant, upon the residual load, the residual loads were also calculated using this value for some cases. These cases are for the largest value of the applied load only. Hence, in Table VII.2 where two sets of values are shown for the largest applied load, the last is for k'_{F_O}. It is seen that, for these fasteners, very little difference in residual loads is predicted from that obtained when k_{F_O} is used.

The predicted loads listed were obtained from the computer routines presented. The predicted loads shown for Specimen I-E were not made using the suggested diffusion method; hence, they would be expected to be somewhat larger than the test results.

By comparing the tabulated test and predicted values the following can be seen.

(1) The largest value of fastener load is seen to occur at the end fastener, as predicted, in nearly all cases. The magnitude of this load is in reasonably close agreement with the predicted value, in general.

(2) The maximum load developed in the doubler is in general, fairly close to the predicted value. The variations are both above and below the predicted values for various specimens.

(3) The values of the fastener loads are seen to be consecutively smaller in the second and third fasteners of the various specimens, in general. There is considerably less agreement between the test and the predicted values in these cases, however.

* Although it is not believed that the bending stresses are large, they would be more significant in the cases of single lap specimens. An analysis as suggested in Article VI.5. would be helpful, but was not carried out.

There is one major factor that affects the test results, the initial slop. Although care was taken so that a sliding fit could be obtained by careful reaming of the holes, it is apparent that some significant slop is present in some of the holes. In general, when a hole is "sloppy" a lesser load will be developed there, and the fasteners adjacent to it will be loaded more than when the hole is "tight". In addition, somewhat less load will then be developed in the doubler than when no significant slop is present. Therefore, when a fastener has a considerably larger load than predicted it indicates that a hole near it is probably somewhat "oversize" and the fastener in that hole would be expected to develop less load than predicted. In Table VII.2 the results indicate some significant slop to be present for example, in Spec. I-A1, fastener #1 & 2, Spec. I-A2, fastener #2, Spec. I-B1, fastener #1 and Spec I-D2 fastener #1,2,&3. In the wide base structure test, Spec. I-E, some significant slop appears to be present at fasteners #2 and #3.

Friction is another item affecting results. In general, since it is neglected, it would be expected that the actual (test) loads in the doubler would be somewhat larger than the predicted values. Hence, it should compensate somewhat for small amounts of slop.

Although the tests results vary more than would be desired from the predicted loads, it is believed that they do substantiate the suggested methods of analysis.

c. Splice Assembly Tests

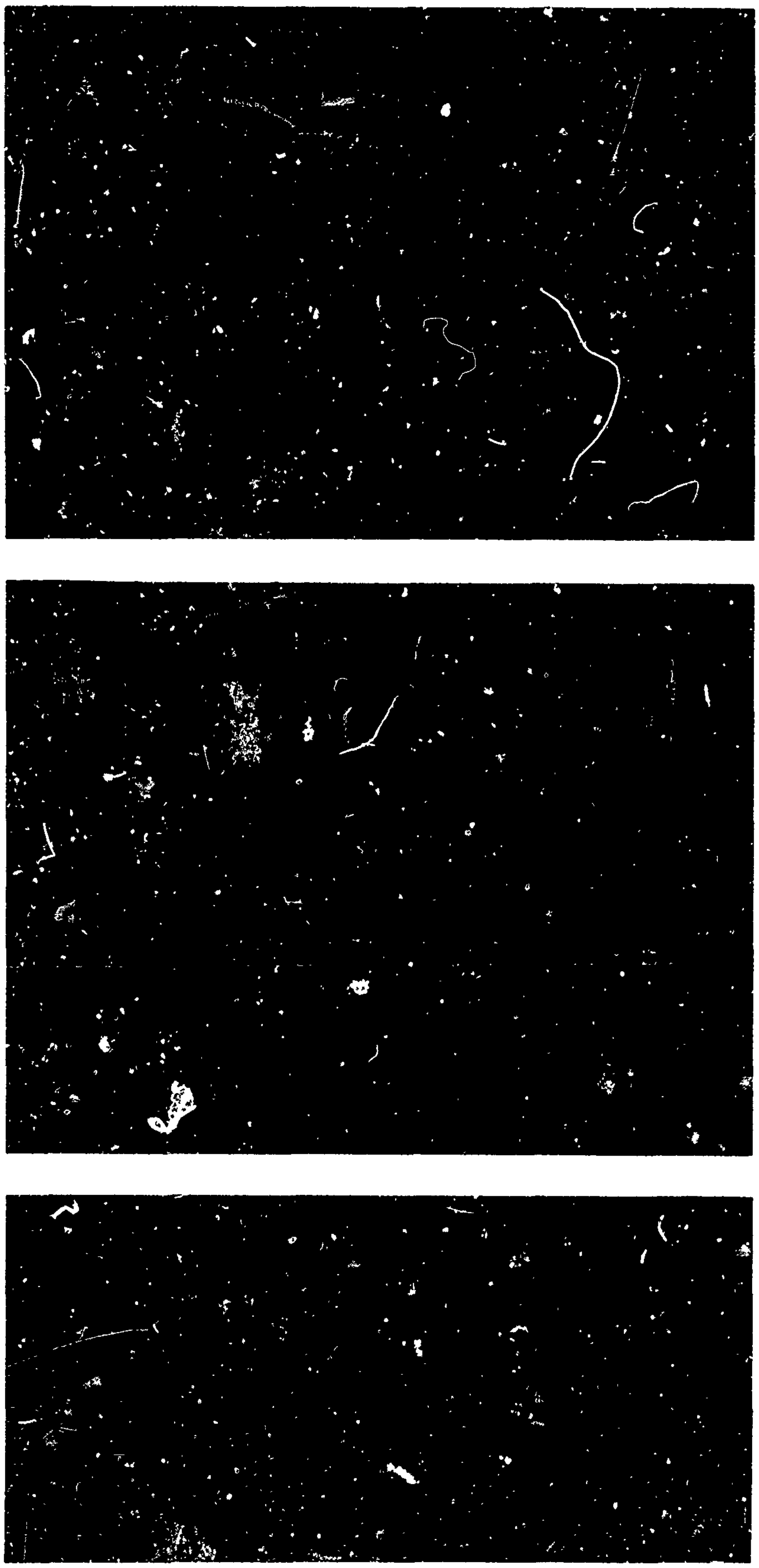
The results of these tests are presented in Table VII.3. For purposes of comparison, the predicted loads are also tabulated. In this case the three (or four) fastener loads at one end are listed. The fastener loads were obtained from the test data in the same manner as described previously for the doubler assembly specimens. The same remarks concerning the factors affecting the doubler fastener loads also apply to the fastener loads in the splice assemblies. In general the agreement between the test and predicted values was not as good as for the doubler assembly specimens. However, the large loads at the end fastener(s) can be clearly seen, and it is believed that the results do substantiate the suggested methods of analysis for the case of splices.

d. Further Notes on Tests

Since, in general, a small amount of slop appeared to be present in many of the specimens, a calculation of the internal loads in Specimen I-A2 was made arbitrarily assuming that fastener #1 was "tight" but that every other (alternate) fastener had .002" slop. That is, half of the fasteners had .002" slop. The resulting predictions showed that

(1) at the applied load Q = 14,152# P_1 would be about 160# larger, P_2 70# smaller and P_3 about 130# larger. Thus, a moderate amount of slop can significantly affect the test results, as far as comparisons with predicted load values are concerned.

(2) at the higher value, Q = 18,000#, there are smaller predicted differences since the slop is less significant.

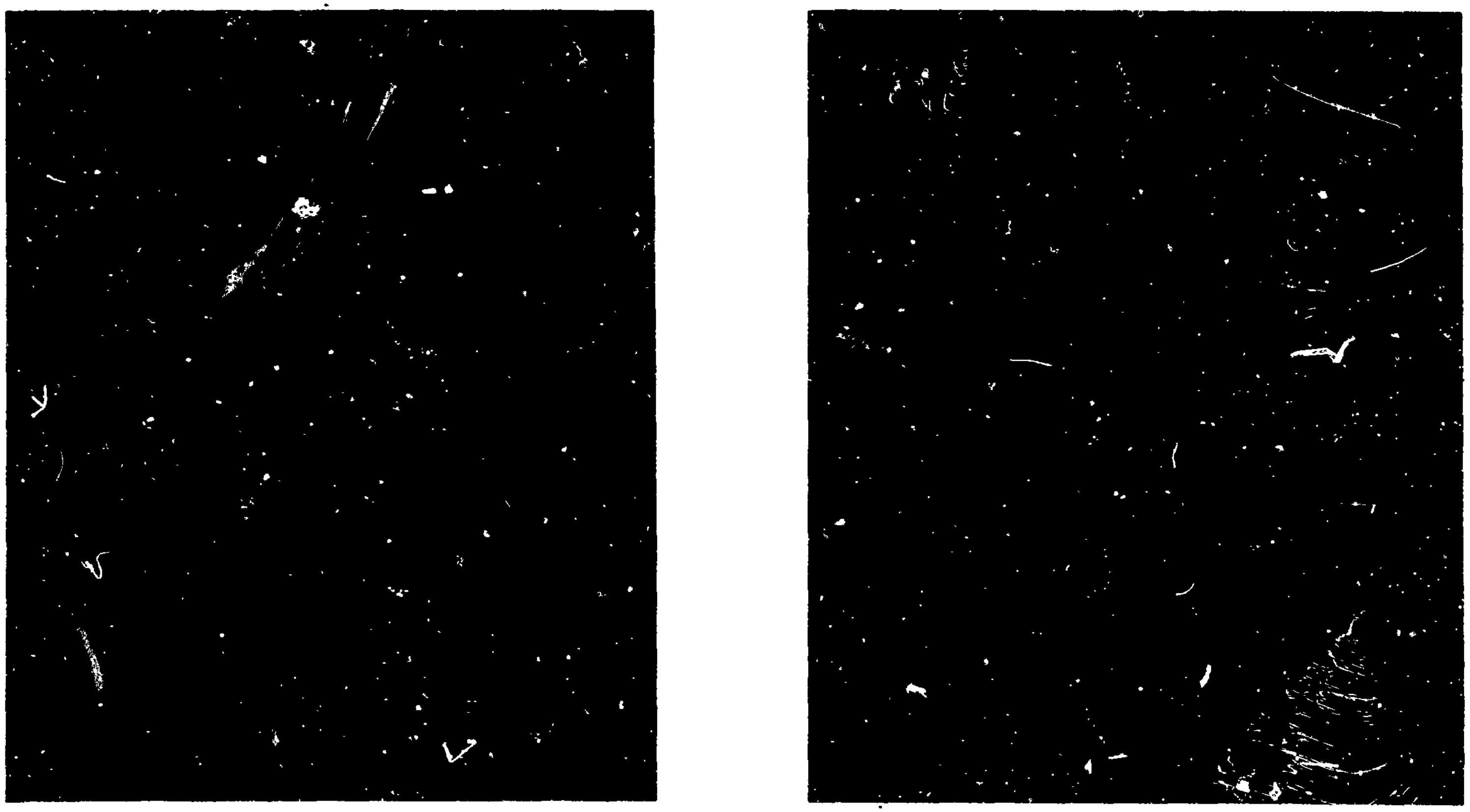


Specimen B
(+P)
(a)

Specimen B
(+P)
(b)

Specimen C
(+P)
(c)

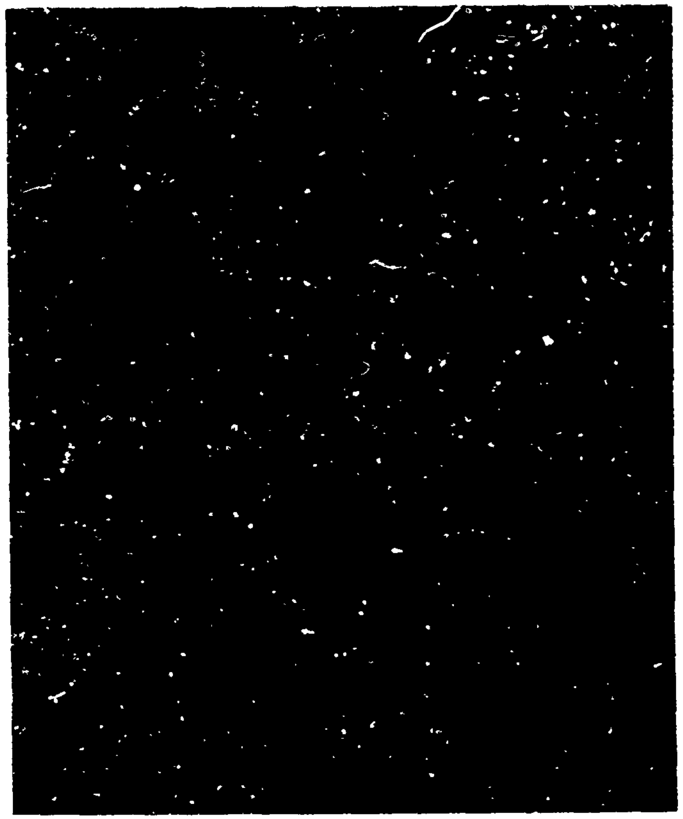
Figure VII.18. Strain Distribution in Photostress Plastic Specimens



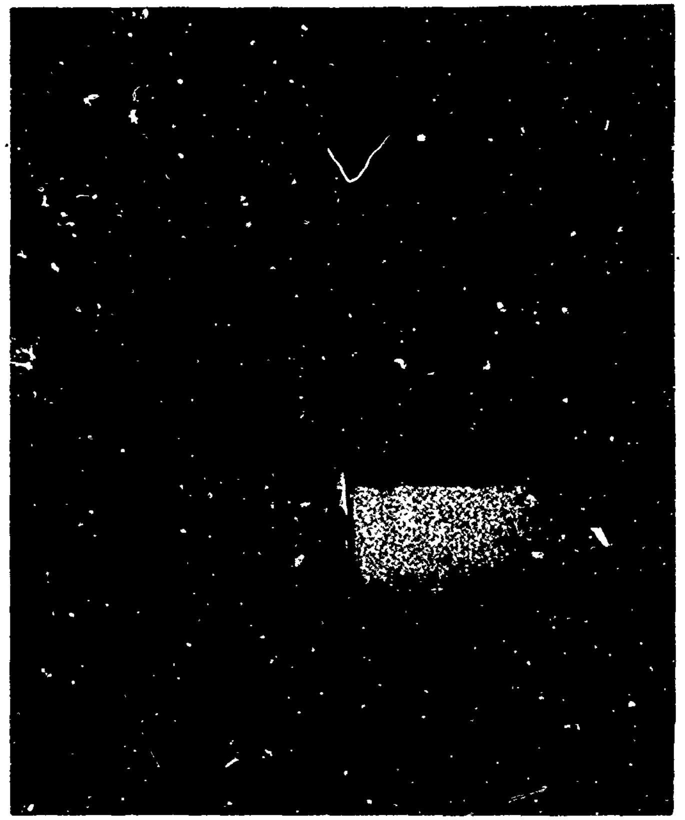
Applied Load = 8150 lbs.
(a)

Applied Load = 12000 lbs.
(b)

Figure VII.19. Strain Distribution in Specimen II-A.2 (Doubler)

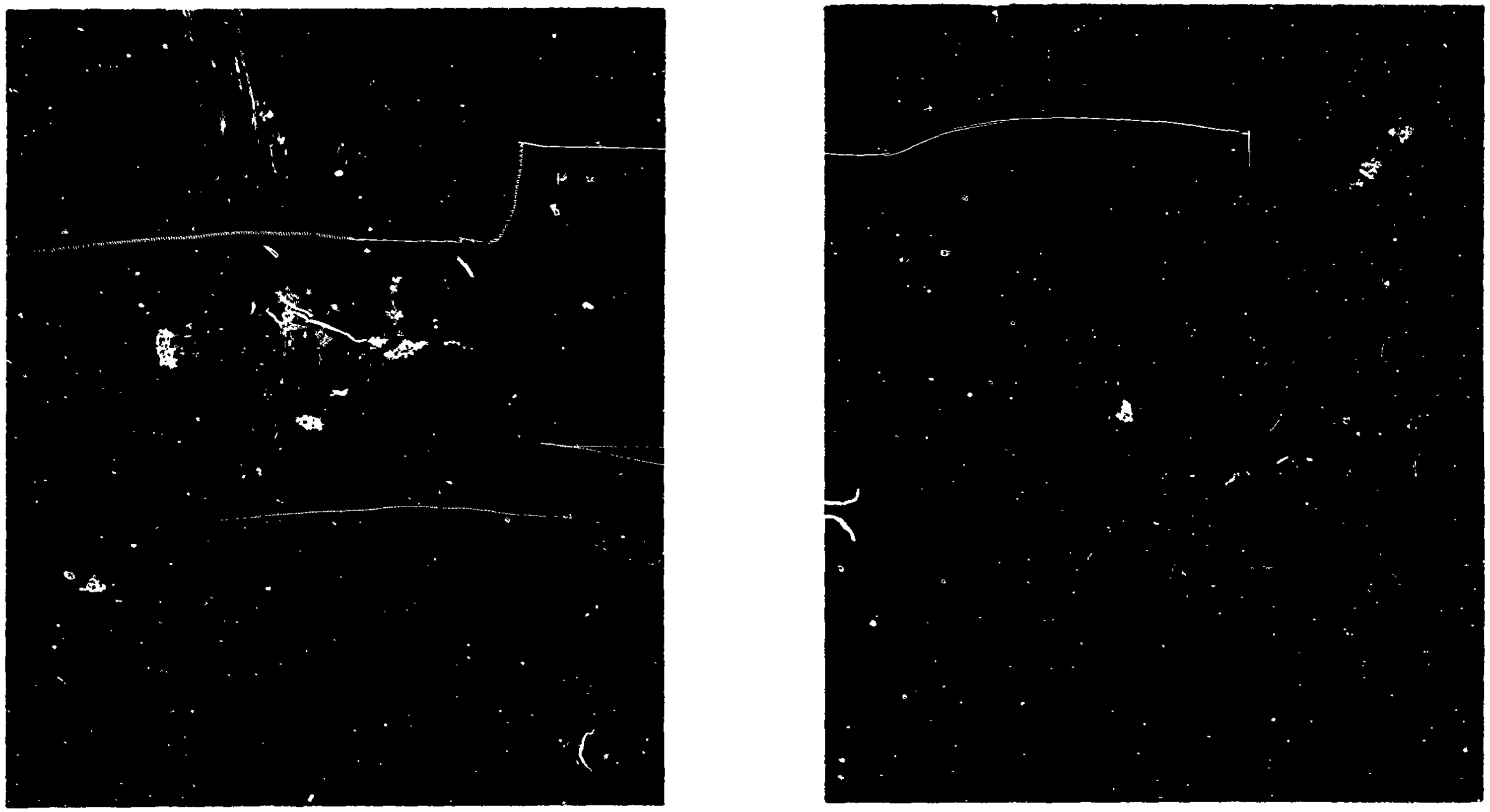


Applied Load = 11,870 Lbs.
(a)



Applied Load = 18,000 Lbs.
(b)

Figure VII.20. Strain Distribution in Specimen I-D1 (Tapered Doubler)



Applied Load = 9320 Lbs.
(a)

Applied Load = 18,000 Lbs.
(b)

Figure VII.21. Strain Distribution in Specimen II-C1 (Tapered Splice)

TABLE VII.2

COMPARISON OF TEST AND PREDICTED INTERNAL LOADS FOR DOUBLER ASSEMBLY SPECIMENS

SPECIMEN	APPLIED LOAD Q	INTERNAL LOADS: LOAD IN FASTENER # 1		RESIDUAL LOAD IN FASTENER # 1		LOAD IN FASTENER # 2		RESIDUAL LOAD IN FASTENER # 2		LOAD IN FASTENER # 3		RESIDUAL LOAD IN FASTENER # 3		LOAD IN FASTENER # 4		RESIDUAL LOAD IN FASTENER # 4		MAXIMUM DOUBLER LOAD	
		TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PRED.
I-A1	7,120	665	750	nil	0	335	428	--	0	255	236	--	0	--	118	--	0	1450	1567
	10,913	1020	1125	nil	-25	540	665	--	10	370	368	--	6	--	183	--	3	2200	2397
	18,000	1240	1544	-410	-352	913	1178	--	97	690	695	--	97	--	346	--	48	3220	3868
*	"	"	"	"	-467	"	"	--	75	"	"	--	106	--	"	--	61		
I-A2	9,207	835	750	nil	0	325	508	--	0	370	343	--	0	--	232	--	0	2260	2233
	14,152	1270	1125	nil	-28	560	786	--	6	600	535	--	7	--	361	--	5	3320	3429
	18,000	1530	1343	-194	-123	690	1012	--	20	680	705	--	33	--	476	--	22	4000	4355
*	"	"	"	"	-216	"	"	--	-12	"	"	--	32	--	"	--	35		
I-B1	6,763	268	1000	nil	0	837	633	--	0	505	401	--	0	--	254	--	0	1953	2672
	12,297	1415	1750	-120	-68	1075	1161	--	9	840	751	--	22	--	475	--	14	4075	4856
	18,000	2260	2232	-210	-430	940	1761	--	75	830	1197	--	130	--	757	--	82	4920	7094
*	"	"	"	"	-553	"	"	--	44	"	"	--	139	--	"	--	106		
I-B2	5,665	1180	1000	nil	0	0	250	--	0	20	158	--	0	214	321	--	0	1960	2226
	11,239	2320	1750	-82	88	80	799	--	-254	80	506	--	-161	720	541	--	120	4200	4426
	18,000	2240	2267	-525	-394	1020	1555	--	-131	260	1035	--	-32	720	879	--	204	5780	7079
*	"	"	"	"	-518	"	"	--	-162	"	"	--	-23	"	"	--	227		
I-C	8,658	760	750	nil	0	630	548	--	0	320	411	--	0	--	308	--	0	2760	3340
	13,293	1100	1125	-500	-27	930	842	--	1	490	638	--	7	--	477	--	4	4400	5127
	18,000	1290	1401	-500	-159	1170	1153	--	14	880	883	--	28	--	667	--	28	5320	6935
*	"	"	"	"	-248	"	"	--	-26	"	"	--	13	--	"	--	24		
I-D1	6,537	936	1000	nil	0	428	681	--	0	636	477	--	0	--	334	--	0	3760	3710
	11,870	1600	1750	nil	-66	1160	1235	--	-1	1260	889	--	22	--	619	--	13	7320	6734
	18,000	2000	2257	-330	-497	1560	1958	--	84	1420	1405	--	91	--	1010	--	91	8360	10202
*	"	"	"	"	-614	"	"	--	44	"	"	--	84	--	"	--	98		

* Values in this row calculated using the "secondary" fastener spring constant, k'_{FO} to show the effect on the residual loads predicted.

TABLE VII.2

(Continued)

SPECIMEN	APPLIED LOAD Q	INTERNAL LOADS: LOAD IN FASTENER # 1		RESIDUAL LOAD IN FASTENER # 1		LOAD IN FASTENER # 2		RESIDUAL LOAD IN FASTENER # 2		LOAD IN FASTENER # 3		RESIDUAL LOAD IN FASTENER # 3		LOAD IN FASTENER # 4		RESIDUAL LOAD IN FASTENER # 4		MAXIMUM DOUBLER LOAD	
		TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED
I-D2	6,520	800	1000	nil	0	340	682	--	0	180	481	--	0	296	340		0	2560	3547
	11,890	1700	1750	nil	-74	580	1270	--	26	460	892	--	16	640	629		9	5280	6466
	18,000	2000	2251	-540	-510	1560	2075	--	192	1420	1391	--	63	680	1011		73	8040	9770
*	"	"	"	"	-626	"	"	--	154	"	"	--	58	"	"		82		
I-E	18,947	1280	1000	nil	0	300	679	--	0	300	460	--	0	--	312	--	0	1950	2990
	34,516	2160	1750	nil	-72	920	1234	--	-2	800	862	--	24	--	583	--	16	4020	5441
	60,000	3000	2358	-792	-809	1320	2082	--	-67	1320	1672	--	214	--	1178	--	191	5920	9378
*	"	"	"	"	-963	"	"	--	-121	"	"	--	211	--	"	--	211		
I-F	12,460	1340	850		--	300	531			240	336				212			2400	2154
	18,000	1960	1234			460	771			380	488				308			3600	3126
I-G1	3,802	410	400	nil	0	158	233		0	212	136		0	--	79		0	1210	952
	6,996	650	700	nil	-36	275	440		10	455	262		11	--	153		6	2000	1751
	13,548	970	1200	-300	-225	220	857		25	690	550		64	--	340		56	3260	3386
*	"	"	"	"	-264	"	"		19		"		70	--	"		65		
I-G2	3,637	800	700	nil	0	460	366		0	160	191		0		100		0	1604	1463
	7,332	1980	1400	nil	-11	480	743		5	250	389		3		203		2	3360	2949
	15,889	**	2700	-350	-358	**	1740		141	**	936		100	**	493		56	**	6389
				"															
I-H1	6,281	1300	1000	nil	0	270	605	--	0	240	366	--	0	246	222	--	0	2320	2501
	14,524	2610	2250	nil	-62	810	1401	--	2	660	871	--	24	520	527	--	14	5000	5781
	18,000	3380	2606	-310	-260	1020	1795	--	60	780	1122	--	72	720	685	--	50	6230	7159
*	"	"	"	"	-294	"	"	--	53	"	"	--	76	"	"	--	58		
I-H2	5,189	970	1000	nil	0	0	211	--	0	0	128	--	0	490	298	--	0	1824	2057
	13,487	2920	2250	nil	103	0	1039	--	-261	320	629	--	-158	520	601	--	125	4660	5360
	18,000	3790	2710	-165	-155	0	1549	--	-186	140	964	--	-86	920	803	--	168	6200	7150
*	"	"	"	"	-190	"	"	--	-193	"	"	--	-82	"	"	--	176		

* Values in this row calculated using the "secondary" fastener spring constant, k'_{F_0} to show the effect on the residual loads predicted.

** Spotwelds failed during loading

TABLE VII.3

COMPARISON OF TEST AND PREDICTED INTERNAL LOADS FOR SPLICE ASSEMBLY SPECIMENS

SPECIMEN	APPLIED LOAD Q	INTERNAL LOADS: LOAD IN FASTENER # 1		RESIDUAL LOAD IN FASTENER # 1		LOAD IN FASTENER # 2		RESIDUAL LOAD IN FASTENER # 2		LOAD IN FASTENER # 3		RESIDUAL LOAD IN FASTENER # 3		LOAD IN FASTENER # 4		RESIDUAL LOAD IN FASTENER # 4	
		TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED	TEST	PREDICTED
II-A1	3,329	705	750	nil	0	305	510	--	0	590	404	--	0	--	404	--	0
	5,618	1,250	1,240	nil	-26	610	870	--	9		699	--	17	--	699	--	17
	11,913	**	2,125	**	-559	**	1,976	--	150	**	1855	--	409	--	1855	--	409
*	"	**	"	**	-659	**	"	--	177	**	"	--	482	--	"	--	482
II-A2	4,802	1,000	750	nil	0	350	533	--	0	110	387	--	0	--	291	--	0
	8,154	2,190	1,240	nil	-35	350	905	--	-1	450	669	--	12	--	503	--	9
	12,000	2,740	1,656	-294	-218	930	1,365	--	32	1070	1013	--	46	--	781	--	54
*	"	"	1,656	"	-331	"	"	--	0		"	--	59	--	"		89
II-B1	4,533	750	1,000	nil	0	320	630	--	0	360	400	--	0	--	260	--	0
	8,224	1,950	1,750	nil	-64	470	1,142	--	0	520	750	--	24	--	488	--	15
	12,000	2,940	2,231	-490	-416	980	1,748	--	82	580	1,164	--	105	--	778	--	89
*	"	"	"	"	-532		"	--	58	"	"	--	122	--	"	--	121
II-B2	3,793	980	1,000	nil	0	24	249		0	116	158		0	380	327		0
	7,525	2,000	1,750	nil	90	280	795		-250	100	505		-159	1180	552		120
	12,000	2,840	2,266	-520	-381	520	1,542		-124	640	1029		-31	720	894		204
*	"	"	"	"	-498	"	"		-148	"	"		-14	"	"		237
II-C1	5,521	870	750	nil	0	550	608	--	0	750	514	--	0	--	444	--	0
	9,324	1,430	1,240	nil	-26	800	1,021	--	-6	1400	873	--	5	--	764	--	14
	18,000	2,045	2,360	-850	-85	1475	1,982	--	-1	2820	1,694	--	19	--	1482	--	33
*	"	"	"	"	-173	"	"	--	-21	"	"	--	37	--	"	--	79
II-C2	5,555	744	1,000	nil	0	1460	728	--	0	860	550	--	0	--	414	--	0
	10,039	1,360	1,750	nil	-57	2280	1,312	--	-3	2020	992	--	-2	--	767	--	18
	18,000	1,630	2,416	-720	-825	3090	2,241	--	-117	2840	2,026	--	244	--	1542	--	199
*	"	"	"	"	-931	"	"	--	-143	"	"	--	256	--	"	--	236
II-D	2,655	1,016	1,000	nil	0	880	782	--	0	765	872	--	0	ONLY 3 FASTENERS IN SPLICE			
	6,021	3,000	2,250	nil	-18	840	1,792	--	18	2180	1,978	--	0	"			

* Values in this row calculated using the "secondary" fastener spring constant k'_{F0} to show the effect on the residual loads predicted.

** Photostress plastic bond failed

SECTION VIII

PRACTICAL APPLICATIONS

VIII.1 INTRODUCTION

The general reasons for which a doubler or a splice installation and analysis might be necessary have been discussed in Section I. As listed there, these include the purposes of improving strength, stiffness and fatigue life necessitated by reasons involving design, service useage or salvage and repair. The purpose of this section is to illustrate some main design points and possible installations, including a suggested general procedure for designing a doubler.

In general, the design of a doubler will have the following basic requirements:

a. Be of such a configuration as to "pick-up" enough load either to properly relieve the base structure, or to stiffen it as required. The amount of load to be picked-up by the doubler must be defined before the doubler design and analysis can be commenced.

b. Accomplish this function without overloading any of the fasteners attaching it. That is, each fastener will have some maximum load that must not be exceeded, established by either a yielding or strength or fatigue consideration. These maximum loads for the fasteners are referred to as the fastener "allowable" loads and are of three principal types

   (1) The fastener load that produces yielding of the fastener-sheet combination. The definition of yielding is presented in Reference (9) along with specific values for numerous fastener-sheet combinations.

   (2) The fastener load that produces static failure of the joint. These loads are presented in Reference (9) for numerous fastener-sheet combinations.

   (3) The fastener load that produces such a bearing stress on either sheet as to begin to reduce the fatigue life of the sheet below its required amount. Or, stated another way, the fastener load that

produces the maximum bearing stress on either sheet that is permissable from the standpoint of the required fatigue life of the sheet. This bearing stress should include any "peaking" effects at the edges of the sheet. These peaking effects will be larger in the case of single shear joints than for double shear joints.

This fatigue consideration may be quite important in the design of doublers and splices. As is well known, available data (Reference 8) shows that **the fatigue life of an axially loaded member** is a function not only of the tension stress, f_t, but also of the bearing stress, f_{br}, in any loaded hole in the member. The larger the ratio f_{br}/f_t, the shorter becomes the fatigue life for repetitive cycles of the loading. Reference 8 shows, for example, that for the case of an applied loading (producing f_{br} and f_t) cycling between $f_{t_{min}} = 0$ and $f_{t_{max}} = 47,000$, the fatigue life for 7075-T6 Alc. sheet will decrease from 10,000 cycles when $f_{br_{max}} = 0$ to 2,400 cycles when $f_{br_{max}} = 47,000$ ($= f_{t_{max}}$). This is, of course, a most significant reduction in fatigue life. Although the data of Reference 8 is for a bearing stress distribution corresponding to a double shear application (obtained by using a pin for applying the bearing loads) it appears to be "useable" for typical single shear applications where some clamp-up is present. Typical examples would be driven rivets or torqued nut installations. Therefore, it is important to consider these possible harmful effects of large fastener loads when a doubler or splice is designed.

In the case of a splice the same basic requirements would be present, except that the load to be transferred is all that must be defined, in VIII.1a.

VIII.2 GENERAL GUIDES FOR DOUBLER DESIGN

The design of a doubler installation is, thus, a tailoring process to satisfy these requirements. The doubler's planform and thickness profiles and the types and numbers of fasteners are the main variables. Space limitations are also a frequent factor. The designing is essentially a "cut-and-try" procedure, using the following general guides.

a. To increase the load picked up by the doubler

(1) increase the doubler planform width
(2) increase the doubler thickness
(3) increase the length of the doubler
(4) increase the number of fasteners
(5) increase the size of fasteners

(6) use stiffer fasteners (material change)
(7) use stiffer doubler material

b. To reduce the "peaking effect", that is the large fastener loads developed at the ends of the doubler

(1) taper the doubler planform
(2) taper the doubler thickness
(3) use a narrower doubler width at the end.
(4) use more flexible (or smaller) fasteners at the ends

c. In order to insure all fasteners loading up efficiently, and also more consistent results, the doubler should be installed (ideally) using close tolerance or reamed holes when non-hole filling fasteners are used. In most practical cases, fasteners of this type will be used since the stiffer steel fasteners are much more efficient in "picking-up" load. In instances where this cannot be done the effects of any possible "slop" should be considered by including this in the analysis.

An inspection of the predicted loads for the various assemblies of Table VII.2 reveals how changing some of these parameters affects the distribution of fastener loads and the load developed in the doubler or splice members.

VIII.3 GENERAL GUIDES FOR SPLICE DESIGN

The main effort is to keep the length of the splice as short as possible. Within this limit the "peaking effect" can be dealt with as outlined in VIII.2b previously. The comments in VIII.2c also apply to splices.

VIII.4 GENERAL PROCEDURE FOR DESIGNING A DOUBLER

The following steps would normally be taken in designing a doubler installation.

a. Define the general area of the base structure that requires reinforcing. This will determine whether the analysis must be made for all of the base structure (a conventional analysis) or for only a part of the base structure (a "wide base structure" analysis) which is somewhat more laborious. Two such cases are illustrated in Figure VIII.1 which shows the need for a doubler on the lower (tension) skin at the root of a swept wing (a) and (b) and on a straight wing, (c).

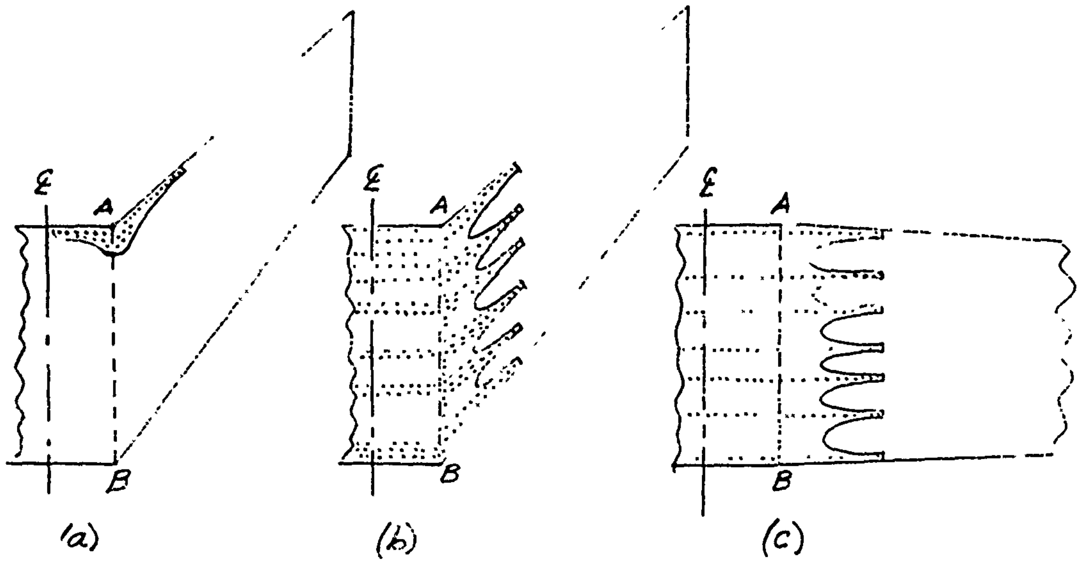


Figure VIII.1 Doubler Installation On A Wing Skin

In (a) the internal structural arrangement and the loads are such that a reinforcement of the skin is necessary only locally, within a few inches of the point A. Hence, the doubler is local on the skin and the "wide base structure" analysis is applicable.

In (b), and in (c), the situation is such that a doubler is required along the entire root chord, AB, and also across the entire root section. Hence a set of doublers, or a single "finger" doubler arrangement is required. Such a doubler is the same as several separate ones but made as an integral unit. The fingers may be required instead of a single edge in order to keep the load from building up too rapidly ("peaking") at the ends of the doubler. That is, the amount of taper that can be put in thicknesswise will usually not be enough in itself to reduce this peaking sufficiently. In Cases (b) and (c) the wide base structure analysis is not required.

(b) Sketch in a doubler over the critical area to be reinforced and extend it beyond this area in order to pick up the load that is to be kept out of the critical area, as in Figure VIII.2.

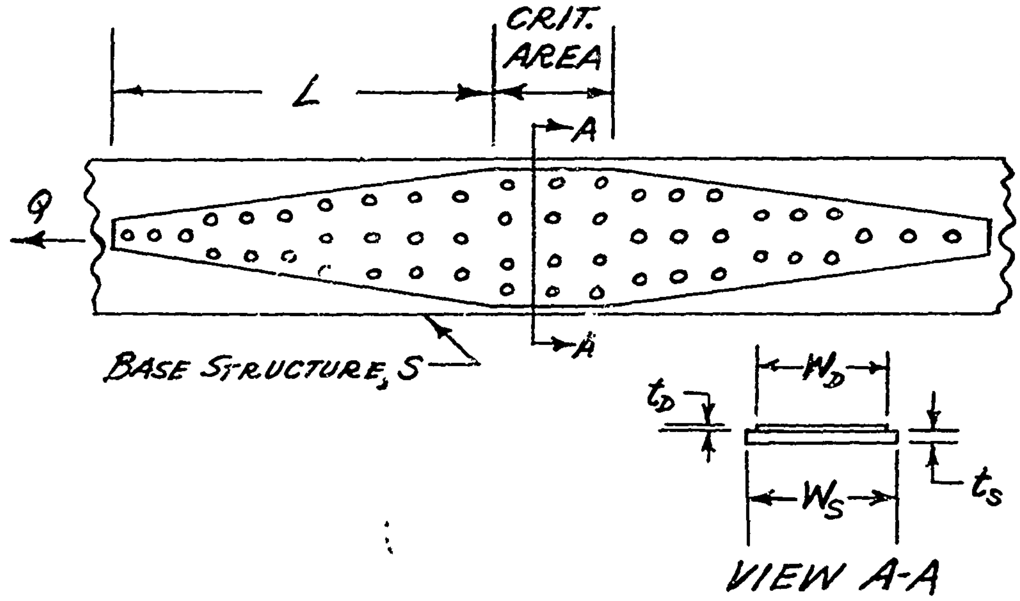


Figure VIII.2 A Preliminary Doubler Installation

c. Obtain a first guess for the required size of the doubler in the critical area (View A-A) as follows:

(1) Assuming the doubler to be, say, 90% efficient in picking up the required load, the load in the doubler at the critical section will then be given as

$$P = Q\left(\frac{.90 A_D E_D}{.90 A_D E_D + A_S E_S}\right)$$

$$= Q\left(\frac{.90 W_D t_D E_D}{.90 W_D t_D E_D + W_S t_S E_S}\right)$$

(2) The required value for P is known, since this is the amount by which the doubler must relieve the base structure. Also the values W_s, t_s, E_s and E_D are known. Hence the required area of the doubler, $W_D t_D$, can be initially estimated as

$$W_D t_D = \frac{P}{.90(Q-P)}\left(\frac{W_S t_S E_S}{E_D}\right)$$

W_D should be about as wide as the base structure* but it could be made smaller

*This is for the case of narrow base structures. For wide base structures the doubler width is, of course, much smaller as in Fig. VIII.1a.

particularly if the resulting thickness, t_D, is judged to be too thin. However, the thinner the doubler the less the eccentricities involved (smaller secondary bending moments) and the better is the structural system in this respect.

d. Next a value for L must be assigned. This should be as short as possible from weight consideration, but must be enough to pick up the required load P and still not generate too great loads at the ends. (as discussed in Art. VIII.1). This can be determined accurately only by a "cut and try" procedure, but as a first guess L can be taken as about 5 times W.

e. A tapered planform for the doubler can then be sketched in, wide enough at the ends to pick up one fastener. (The end fastener load can be initially guessed at using the suggested formula in Article III.2, to estimate the required size of fastener.)

f. An array of fasteners can then be located as shown in Figure VIII.2. In order to pick up load efficiently the fastener-sheet combination must have a reasonably stiff joint spring constant, k_F. This usually means that steel fasteners are required. However, if aluminum fasteners are used the diameter should be large enough that the joint is critical in bearing, not in shear, to insure a ductile joint rather than a brittle one. In any event the load-deflection characteristics for the fasteners selected must be available.

g. An analysis can now be made as discussed in Section III to determine the internal loads. In most practical cases the simple analysis of Art. III.2, and Table III.1 is adequate. The resulting internal loads must be such that

(1) The resulting load (or stress) in the base structure is reduced to a satisfactory magnitude to satisfy any strength, stiffness or fatigue requirements.

(2) The load in the doubler is satisfactory. That is, the stress levels in the doubler (and, hence, the values of E_t used for the doubler

element spring constant determinations) are consistent with what was assumed in the analysis, normally elastic stress levels.

(3) The local bearing stresses due to the fastener loads are low enough so as not to fail to meet the fatigue life requirements when the base structure and the doubler are in tension.

h. If the load in the base structure is not found to be sufficiently reduced (doubler load is not large enough) some or all of the steps in Article VIII.2 are required. Opposite steps are, of course, taken if the doubler load is found to be larger than necessary, to keep the weight down.

i. If the peaking effect at the ends is too large a reshaping in this vicinity is required as sketched in Figure VIII.3. The initially guessed shape is shown as the dashed lines. The final shape (arrived at by "cut and try") is shown by the solid lines. Note that the ends may be tapered in thickness to keep the end fastener loads small enough.

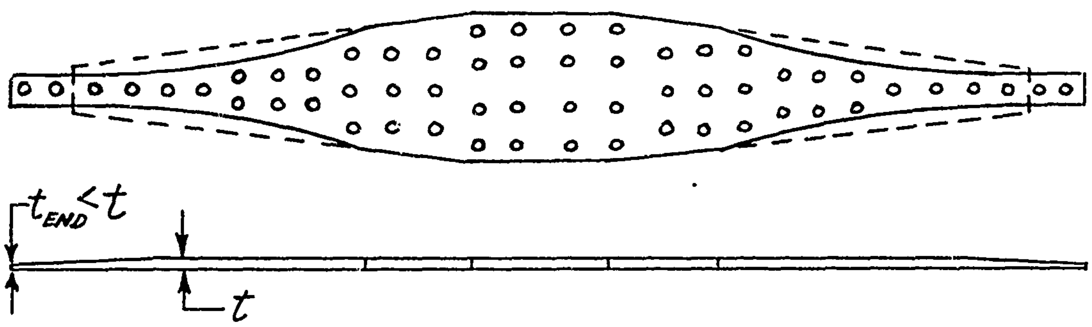


Figure VIII.3 A Tapered Doubler

Summarizing, the final doubler design is arrived at by the "cut and try" procedure, using the previously outlined steps and engineering judgement as a guide in making successive trials. The final design must satisfy all strength, stiffness and fatigue criteria for the structure. In most practical cases the usual requirement of no significant yielding at limit load means a simple elastic analysis (as in Table II.1 or III.2). If each type of joint is ductile (critical in bearing) the design should then

present no problems in carrying the ultimate load.* That is, a plastic analysis at the ultimate load factor should not usually be necessary in such cases, but it can be made as suggested in this report. Any detrimental secondary effects should be considered, as suggested in Article VI.5.

Some additional comments on this subject are included in Appendix I.

The design of a splice would be approached in the same manner when there are many rows of fasteners. That is, the thickness profile would be tapered to keep the peaking effect as small as necessary from any strength, yielding or fatigue considerations.

*When there are only a few fasteners present, which is the usual case for splices, the plastic analysis for the ultimate load is more likely to be necessary.

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APPENDIX I

ADDITIONAL TOPICS AND METHODS

AI.1 INTRODUCTION

The purpose of this appendix is to present additional methods, discussions and illustrative examples which, for purposes of clarity, have not been included in the previous sections of the report. The following topics, by article number, are included.

AI.2 "Short-Cuts" For Symmetrical Doubler and Splice Installation.

AI.3 Accounting For The Effect of "Slop" and Plasticity on Internal Loads.

AI.4 Accounting For the Effect of "Slop" and Plasticity on Residual Loads.

AI.5 Accounting For "Slop" at One Or More Fasteners In a Row or Group.

AI.6 Doublers on Wide Base Structures

AI.7 Doublers Reinforcing A Cut-Out

AI.2 SHORT-CUTS FOR SYMMETRICAL DOUBLERS AND SPLICES

When symmetry is present in both the structure and in the applied loads it is not necessary to calculate all of the fastener loads as in Table III.1 and III.2. This can save considerable time and chance for error in a hand analysis. The analyses can be shortened as follows:

a. Structure having an even number of fasteners, N.

(1) Doubler Calculations

The two center fasteners, $n = N/2$ and $n = N/2 + 1$ must have equal and opposite loads. Hence it is only necessary to include $N/2 + 1$ fasteners in the table of calculations. The "error" in any trial will then be

$$P_{N/2} + P_{N/2+1} \text{ or } 6_{N/2} + 6_{N/2+1}$$

(2) Splice Calculations

Again, only $N/2 + 1$ fasteners need to

be included. However, in this case the two center fasteners must have equal (but not opposite) loads. Hence the "error" will be $P_{N/2} - P_{N/2+1}$ or $⑥_{N/2} - ⑥_{N/2+1}$

b. Structures having an <u>odd number</u> of fasteners, N.

(1) Doubler Calculations

Only $(N+1)/2$ fasteners need to be included in the analysis. The center fastener, $n = (N+1)/2$ must have no load. Hence the "error" will be $P_{N+1/2}$ or $⑥_{N+1/2}$

(2) Splice Calculations

Only $(N+3)/2$ fasteners need to be included in the analysis. The fasteners on each side of the middle one, $n = (N-1)/2$ and $n = (N+3)/2$ must have equal loads. Hence the "error" will be $P_{(N-1)/2} - P_{(N+3)/2}$ or $⑥_{(n-1)/2} - ⑥_{(N+3)/2}$.

It should be remembered, however, that an unsymmetrical distribution of "slop" destroys the symmetry of an otherwise symmetrical structure. Sometimes, however, a structure which is very nearly symmetrical is considered to be so in order to facilitate a hand analysis and obtain quick estimates.

AI.3 ACCOUNTING FOR THE EFFECT OF "SLOP" AND PLASTICITY ON INTERNAL LOADS

The analysis outlined in Article III.6 does not (as presented) include provision for the presence of "slop" at one or more fasteners. However, this effect can be accounted for by a simple addition to the procedure outlined in Article III.6 and illustrated in Table III.3. It is only necessary to include the effect of "closing up" the slop by including the term $\Delta(\delta_S - \delta_D)_n$ at any fastener, n, subject to slop. The procedure then accounts for the fact that until the slop is "closed-up" the fastener is ineffective (or $k_{F_n} = 0$).

<u>Procedure</u> (Carried out in a table similar to III.3)

a. At any fastener having a specified slop, Δc, include the term $\Delta(\delta_S - \delta_D)$ in Col. ① The value of this

is obtained from Col. ② of the basic table (Table III.1 or III.2) for each unit solution.

b. Then in the analysis include the limiting effects as these clearances are successively closed up and the respective fasteners become effective. That is, for the first increment, $k_{F_n} = 0$ but when the value of $\Delta(\delta_S - \delta_D)_n$ equals the initial slop, Δc_n, the fastener becomes effective, $k_{F_n} \neq 0$, and another unit solution is required for the next loading increment.

c. The previous effects of limits due to plasticity (as in Table III.3) are still present and are considered just as before.

d. It is possible that in some cases the initial slop will not be completely closed up. This would be most likely to occur at the "center area" of a long doubler (or splice). The following example illustrates the procedure.

Example Problem

Rework the example problem of Figure III.11 assuming that there is an initial slop of .005" at fastener #2, #4, #7 and #9. Since the slop is symmetrical, only half of the structure needs to be considered, as n the previous example.

The analysis is carried out in Appendix Table AI.1 which is similar to Table III.3. Note, however, that provision is made in Column 1 for the value of $\Delta(\delta_S - \delta_D)$ at fastener #2 and #4.

a. the first unit solution is made assuming $k_2 = k_4 = 0$ $(=k_7 = k_9)$ because of the slop.

b. the values of $\Delta(\delta_S - \delta_D)$ are entered in Col. ② as obtained in (a).

c. the limiting values of .005", the initial slop, are entered in Col. ③ for these terms. This means that when any slop closes up a "new" structure is present since that fastener becomes effective.

d. Columns ④ - ⑥ are completed as indicated. It is seen that the smallest limiting ratio is due to the slop at fastener #2 closing up.

e. the second unit solution is made having only k_{F_4} (and k_{F_9}) = 0 and columns 7 - 11 are completed. The slop at fastener #4 (and #9) is not yet closed, but fastener #1 goes plastic, limiting this loading increment.

f. a third unit solution having k_{F_1} = 103,300 and k_{F_4} = 0 is made and Col. 12 - 16 are completed. The limit for this increment is due to the slop at fastener #4 finally closing up.

g. a fourth unit solution is made for k_{F_1} = 103,300 and all other fasteners having k_F = 256,000. The limit here is the allowable load for fastener #1 of 6450# (per Figure III.11b). It is seen that this occurs for an applied load of Q_L = 44,205#.

The values of $\Delta(\delta_S - \delta_D)_n$ are accumulated as shown in order to be able to determine the residual loads after the applied load, Q_L = 44205, is removed. This is discussed next.

AI.4 ACCOUNTING FOR THE EFFECT OF "SLOP" IN THE PLASTIC RANGE ON RESIDUAL LOADS

In order to determine the residual loads the procedure of superposition can be used but not as simply as in Article II.7 where slop was not considered. In this case the loading to be superposed on the results of Table AI.1 must be arrived at as follows, referring to Table AI.2.

a. To begin the "unloading" procedure, which uses the applied load for later superposition, all fasteners are effective (as indicated in Col. 21 of Table AI.1. Hence a unit analysis is made for an applied load of Q_L = 44205 and k_{F_1}---k_{F_5} = 256,000, the elastic values. The limiting values of $\Delta(\delta_S - \delta_D)_n$ are shown in Col. 2 since, "working backwards", at these values the fasteners will again become ineffective. These values of $\Delta(\delta_S - \delta_D)$ are obtained by subtracting the initial slop from the values in Col. 21 of Table AI.1. It is seen that fastener #4 is the limiting one, becoming ineffective before fastener #2 does.

b. A second unit solution is then made in which $k_{F_4} = 0$ (and, hence, $P_{F_4} = 0$). The limiting value of $\Delta(\delta_S - \delta_D)_2$ is still .01648" since it has not yet reached this amount. The limiting value of $\Delta(\delta_S - \delta_D)_4$ is shown as .00732", the initial slop, since this represents a return to the original condition (before any loading) The value .00732" is from Col. 21 of Table AI.1 . Actually, because of yielding, the value of $\Delta(\delta_S - \delta_D)_n$ can never reach its limit from Col. 21. Col. 7 through 11 are completed as shown, with fastener #2 now becoming ineffective.

c. A third unit solution is made having $k_{F_2} = k_{F_4} = 0$. The limits for both $\Delta(\delta_S - \delta_D)_4$ and $\Delta(\delta_S - \delta_D)_2$ are now from Col. 21 of Table AI.1. The final results are shown in Col. 16.

The residual loads are obtained by superposition, subtracting the values of Col. 16 Table AI.2 from those in Col. 21 Table AI.1. It is seen that because of yielding at fastener #1, the "slop" at fastener #2 and #4 does not return to its original value of .005", but remains partially closed-up. Hence, any future analyses (having Q_L less than 44,205#, the allowable amount in this structure) would start from this basis. That is they would be simple elastic analyses made as in Table III.1 or III.2 but would have initial slop values included for the fasteners #2 and #4 of the amount

$$\Delta c_2 = .00500 - .00298 = .00202" \; (=\Delta c_9)$$

$$\Delta c_4 = .00500 - .00114 = .00386" \; (=\Delta c_9)$$

The analysis would be made as in Table AI.1, the limits in Col. 3, 8 etc. being either these "net slop" values or the values of Q applied. The results would then be added to the residual loads to obtain the final values, just as in Table III.7.

TABLE AI.1

DETERMINATION OF INTERNAL LOADS IN THE PLASTIC RANGE WHEN INITIAL SLOP IS PRESENT

①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫	⑬	⑭	⑮	⑯	⑰	⑱	⑲	⑳	㉑	㉒
ITEM	FIRST UNIT SOLUTION	LIMITING LOAD LEVEL	POSSIBLE LIMITING RATIOS	FIRST LOADING INCREMENT	SUM OF LOADS	SECOND UNIT SOLUTION	LIMITING LOAD LEVEL	POSSIBLE LIMITING RATIOS	SECOND LOADING INCREMENT	SUM OF LOADS	THIRD UNIT SOLUTION	LIMITING LOAD LEVEL	POSSIBLE LIMITING RATIOS	THIRD LOADING INCREMENT	SUM OF LOADS	FOURTH UNIT SOLUTION	LIMITING LOAD LEVEL	POSSIBLE LIMITING RATIOS	FOURTH LOADING INCREMENT	SUM OF LOADS	FIFTH UNIT SOLUTION
			r_1	Δ_1	$\Sigma\Delta$			r_2	Δ_2	$\Sigma\Delta$			r_3	Δ_3	$\Sigma\Delta$			r_4	Δ_4	$\Sigma\Delta$	
	q_1=①	$q_{LIM.}$-q②	③/②	④$_{MIN.}$ x ②	⑤	q_2= q⑧-q⑥	$q_{LIM.}$-q②	(⑧ - ⑥)/⑦	⑨$_{MIN.}$ x ⑦	⑥ + ⑩	q_3= q⑧-q⑪	$q_{LIM.}$-q②	(⑬ - ⑪)/⑫	⑭$_{MIN.}$ x ⑫	⑪ + ⑮	q_4= q⑱-q⑯	q_{LIM}-q②	(⑱ - ⑯)/⑰	⑲$_{MIN.}$ x ⑰	⑯ + ⑳	
	FROM TABULAR ANALYSIS	P_H FROM FIG. IV.11b				FROM TABULAR ANALYSIS	P_H FROM FIG. IV-11b				FROM TABULAR ANALYSIS	P_H FROM FIG. IV-11b				FROM TABULAR ANALYSIS	P_H FROM FIG. IV-11b				
	k_1=k_3=k_5= 256,000; k_2=k_4=0 (i.e. "slop" @ #2 & #4 is .005")					k_1--k_3=k_5= 256,000; k_4=0 (i.e. "slop" @ #4)					k_1= 103,300, k_2=k_3=k_5= 256,000; k_4=0 (i.e. "slop" @ #4)					k_1= 103,300, k_2--k_5= 256,000					
Q_L	44,800	44,800	1.00	8650	8650	36150	44,800	1.00	18,075	26,725	18,075	44,800	1.000	3040	29,765	15,035	44,800	1.000	14,440	44,205	Not Required Since P_{F1} = 6450 # (Allowable Load)
P_{F1}	9,262	4,900	.529	1790	1790	6220	4,900	.500	3,110	4,900	1,625	6,450	.954	273	5,173	1,328	6,450	.962	1,277	6,450	
P_{F2}	0	4,900		0	0	3800	4,900	1.290	1,900	1,900	2,460	4,900	1.219	414	2,314	1,980	4,900	1.310	1,906	4,220	
$\Delta(\delta_5-\delta_4)_z$	.02590"	.005"	.193	.005"	.005"	.01485"	--	--	.00743"	.01243"	.00962"	--	--	.00162"	.01405"	.00774"	--	--	.00743"	.02148"	
P_{F3}	4026	4,900	1.216	777	777	2300	4,900	1.793	1,150	1,927	1,492	4,900	1.206	251	2,178	1,150	4,900	2.37	1,108	3,286	
P_{F4}	0	4,900		0	0	0	4,900		0	0	0	4,900		0	0	594	4,900	8.24	572	572	
$\Delta(\delta_5-\delta_6)_y$	.00929"	.005"	.431"	.00180"	.00180"	.00532"	.005"	.602	.00262"	.00442"	.00345"	.005"	.168	.00058"	.00500"	.00232"	--	--	.00223"	.00732"	
P_{F5}	734	4,900	6.68	142	142	420	4,900	11.33	210	352	272	4,900	17.00	46	398	183	4,900	24.6	176	574	
P_{D5}	14,022	--	--	2710	2710	12750	--	--	6,380	9,090	5,850	--	--	982	10,072	5,225	--	--	5,020	15,102	
P_{B5}	30,778	--	--	5940	5940	23400	--	--	11,705	17,645	12,225	--	--	2058	19,703	9,810	--	--	9,480	29,103	

TABLE AI.2

CALCULATION OF SUPERPOSITION LOADS FOR DETERMINING RESIDUAL LOADS - SAME HEADINGS AS IN TABLE AI.1 ABOVE

	k_1--k_5= 256,000					k_1---k_3=k_5= 256,000, k_4 =0					k_1=k_3=k_5= 256,000, k_2=k_4=0					
Q_L	44,205	44,205	1.000	19,400	19,400	24,805	44,205	1.000	21,300	40,700	3,505	44,205	1.000	3505	44,205	0
P_{F1}	7,490			3,290	3,290	4,270			3,670	6,960	725			725	7,685	0
P_{F2}	4,510			1,980	1,980	2,610			2,240	4,220	0			0	4,220	0
$\Delta(\delta_5-\delta_4)_z$	.01760"	.01648"	.937	.00773"	.00773"	.01018"	.01648"	.860	.00875"	.01648"	.00202"	.02148"	2.47	.00202"	.01850"	0
P_{F3}	2,615			1,148	1,148	1,583			1,360	2,508	317			317	2,825	0
P_{F4}	1,355			594	594	0			0	594	0			0	594	0
$\Delta(\delta_5-\delta_6)_y$	.00529"	.00232"	.438	.00232"	.00232	.00365	.00732"	1.370	.00314"	.00546"	.00072"	.00732	2.58	.00072"	.00618"	0
P_{F5}	418			183	183	289			248	431	58			58	489	0
P_{D5}	16,388			7,195	7,195	8,752			7,518	14,713	1,100			1,100	15,813	0
P_{B5}	27,817			12,205	12,205	16,053			13,782	25,987	2,405			2,405	28,392	0

TABLE AI.3

RESIDUAL LOADS AND SLOP

ITEM	㉑$_{AI.1}$ − ⑯$_{AI.2}$
Q_L	0
P_{F1}	-1235
P_{F2}	0
	.00298"
P_{F3}	461
P_{F4}	-22
	.00114"
P_{F5}	85
P_{D5}	-711
P_{B5}	711

AI.5 ACCOUNTING FOR SLOP AT ONE OR MORE FASTENERS IN A ROW OR GROUP

In Article III.5 and Figure III.9 the grouping of several fasteners in a row into a single larger effective fastener was discussed. If one or more fasteners in a row (or in a group of several rows) is in a "sloppy" hole and if the effect of this is to be evaluated, an additional refinement is required. This uses the principle of superposition of separate analyses as discussed elsewhere and illustrated in Art III.6 and AI.3. The steps are as follows:

a. Assume the sloppy fasteners are "out" or ineffective. Then determine the effective k_F for the remaining fasteners in the group and carry out a unit analysis for the internal loads.

b. Determine the increment of applied load, ΔQ, required to close up the first of any sloppy holes and let this fastener be then considered as fully effective. This increment is calculated as was done in Table AI.1

c. Repeat steps a. and b. until the sum of the increments of the applied loading equal the true applied loading. The internal loads will be the sum of the various increments of internal loads obtained in the successive analyses (as in Table AI.1)

This can be quite an effort if there are numerous groups having varying amounts of slop within the group. In such cases it may be more desirable to simply omit one or more such fasteners from the entire group, assume the remaining ones to be "tight", and thereby avoid the above tedious analysis. This requires some engineering judgement, but it can in many cases be an adequate approach.

AI.6 DOUBLERS ON WIDE BASE STRUCTURES

Such cases would arise where it is necessary to reinforce a skin at a local (or small) area only. This could be due to local structural or loading conditions or cut-outs as discussed in Article AI.6. Such a case could also arise simply because an unrelated member (bracketry) is attached to a skin.

The basic approach has been suggested in Article III.9 However, the results of the tests of the specimen of Figure VII.8 and of separate calculations for "shear-lag" show that it is more reasonable to establish the individual diffusion lines as shown in Figure AI.1 not as in Figure III.15 or III.16.

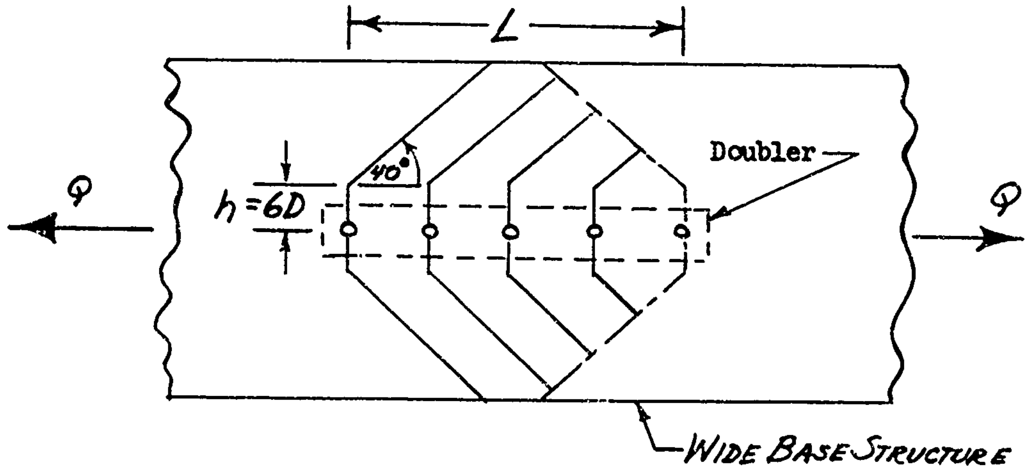


Figure AI.1 Diffusion Lines For Practical Analysis Purposes

That is, as would be expected the dimension h appears to be some function of the rivet diameter and the length L. This function is not known and would need considerable experimental and analytical work to be accurately defined. For purposes of preliminary engineering design the value h = 6D (D = Fastener Diameter) is arbitrarily suggested. The slope of the diffusion lines would also need further experimental effort to be accurately defined. However, the slope of 40°, or perhaps slightly less, seems to be reasonable for arbitrarily defining the effective width for preliminary design purposes. It should be remembered that these arbitrary diffusion lines are being used not to define the local stresses in the sheets, but rather to obtain a more realistic estimate of the fastener loads. There are two consequences here:

a. If the diffusion lines are taken at too steep a slope (a 90° angle is equivalent to considering the base structure fully effective) the fastener loads and the doubler load will be over-estimated.

b. If the diffusion lines are at too shallow an angle the fastener loads and the doubler load will be underestimated.

It is believed that the assumptions of Figure AI.1 give a reasonable compromise. The analyst can, of course, calculate "limiting" cases for a and b above using a lesser slope, say 25°, in b. Then, to be conservative, use a for checking out the doubler and the bearing stresses on the base structure and b for checking out the base structure in its critical area where load relief was originally required.

The predicted loads for Specimen I-E in Table VII.2 were computed assuming all of the base structure to be effective. That is, the suggested diffusion analysis was not made. Hence, it would be expected that the resulting test values of fastener loads and maximum doubler load would be smaller than the predicted values. This is what is seen in the table except at the end fastener, #1, where the test load is larger. It appears that this is partly due to some slop in fastener #2, which makes the results somewhat less clear as to the exact effect of the wide base structure and the associated diffusion effects. However, the total load developed in the doubler is seen to be considerably less in the test results than is predicted by assuming all of the base structure to be effective. This would be anticipated.

The suggested analysis for the case of wide base structures is admittedly arbitrary and much data is needed for making it more accurate. However, such structural arrangements do arise and the designer needs some practical rational procedure for estimating the internal loads for such cases. The suggested approach is made on this basis.

AI.7 DOUBLERS REINFORCING A CUT-OUT FOR AXIAL STRENGTH OR STIFFNESS

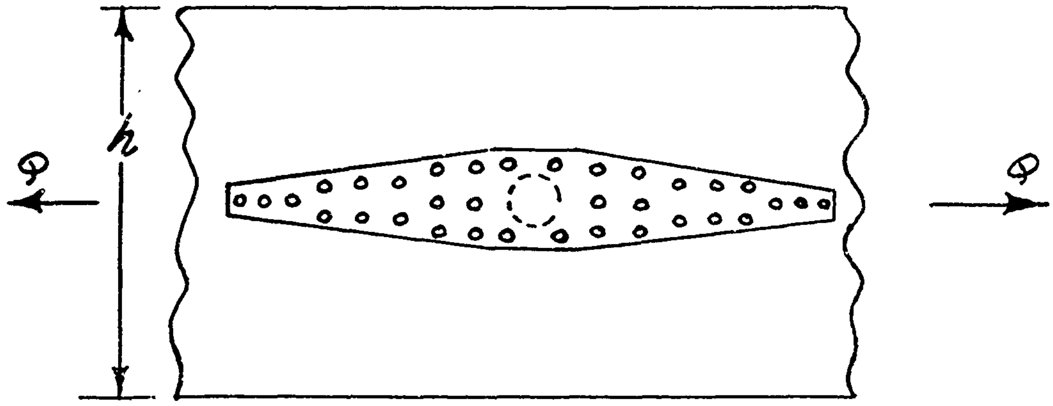
It may be necessary to install a doubler to provide either the strength or stiffness lost in a member because of the presence of a cut-out. (This should not be confused with the reinforcing of a hole from a shear strength or buckling consideration which is another problem). Two general cases are mentioned below. In either case the suggestions of Article VIII.4 and AI.6 apply. In the first case the doubler covers the hole. In the second case the doubler also has the hole.

a. Doubler Covering the Hole

(1) The effective edge of the base structure at the hole is arbitrarily defined by the lines having a 40° slope as shown in Figure AI.2b. These are drawn tangent to the cut-out. The cross-hatched width is ineffective.

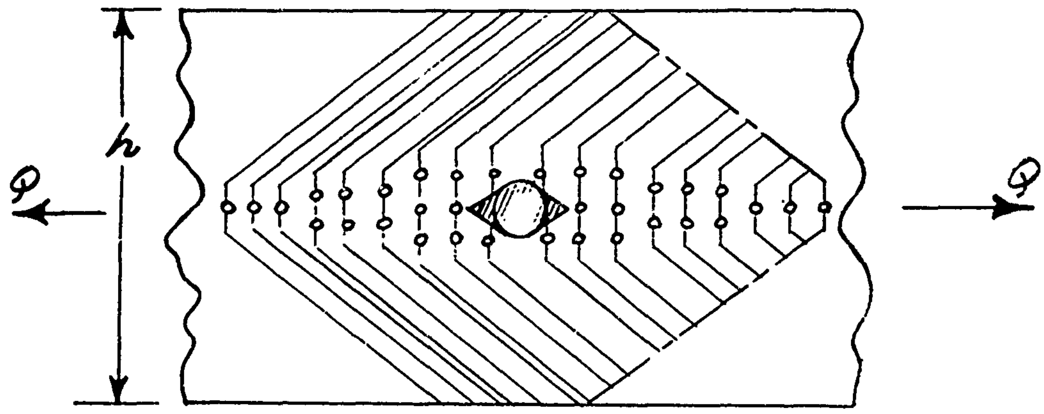
(2) The base structure is then defined by these edges, (1) above, by the diffusion lines shown, and by the outer edges of the base structure if they lie within the diffusion lines (See Art. III. and VIII.4)

(3) An analysis is then carried out to determine the internal loads and the adequacy of the doubler installation as discussed in Article VIII.4.



(a)

Installation Of Doubler



(b)

Effective Base Structure

Figure AI.2 Solid Doubler Reinforcing A Cut-Out

b. Doubler Having The Cut-Out Also

(1) The base structure would be defined as suggested previously.

(2) The effective edge of the doubler in the area of the hole would be defined by the 40° lines as shown in Figure AI.2b. That is, the effective edge of the doubler at the hole would be defined in the same manner as the base structure.

(3) The analysis would then be carried out as described previously.

APPENDIX II

REVERSED LOADINGS

The methods discussed in this report and the specimens tested have been for the case of loads applied in one direction only. The tests were all made for the simpler applied tension load. In practice, the loads may be in either direction.

The methods suggested should also be applicable for the case of successive applied loads that include load reversals. That is, both tensile and compressive loads may be applied in random order. The "bookkeeping" would be more involved, of course, for excursions into the plastic range, particularly when slop is present. However, the basic approach suggested in Appendix I, Article AI.3 could be used. Under the usual circumstances of having no available experimental load-deflection data for "compressive" joint loads, it would be necessary to assume the compressive data to be identical to the tensile data. This is sketched in Figure AII.1 where (+) indicates tensile and (-) indicates compressive loads.

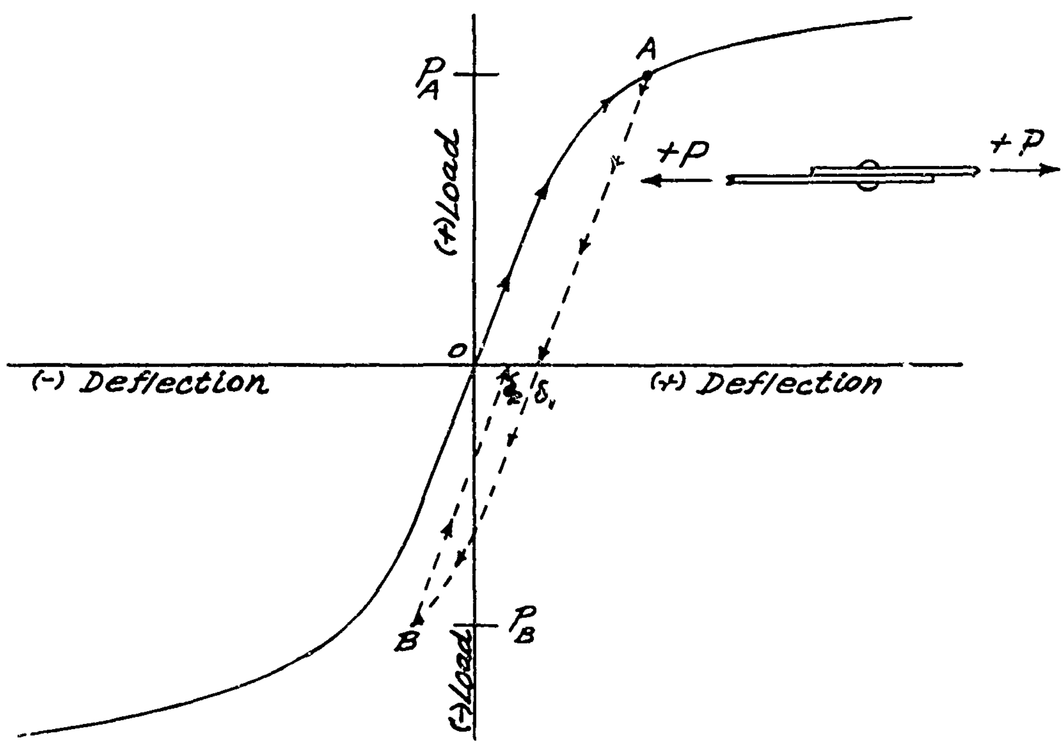


Figure AII.1 Load-Deflection Curve for Reversed Loadings

Under a reversed loading (+ to -), the action could be assumed as follows:

1. Beginning at O, the tensile load causes movement as described by the line OA.

2. When this load is removed, the line $A\delta_1$, is followed, leaving a permanent set, δ_1.

3. When a compressive load is applied, the movement is assumed to be along the line δ_1B which has the same slope as the "compressive" load deflection curve. Actually, it would be expected that this would not occur but that there would be a "transition region" for small values of load (-P) having a considerably lesser slope. This could be defined only by tests and would probably be a function of the specific fastener and sheet combination.

4. When the compressive load is removed the movement would be defined by the line $B\delta_2$, to the permanent set δ_2 etc.

In most practical applications either the tensile or the compressive loadings would be dominant. That is, the reversed loading would be smaller and would not extend into the reversed plastic range. If it did a serious fatigue problem might be anticipated.

Thus, it is seen that attempting to account for the effects of reversed loadings is a difficult task, requiring even more experimental data that is not presently available. However, when the loads are in the elastic range no significant permanent set is generated and only the simpler analyses as in Tables III.1 and III.2 are necessary.

APPENDIX III

ADDITIONAL COMPUTER ROUTINES

AIII.1 INTRODUCTION

The purpose of this appendix is to present additional routines that have been developed for specific installations. These are described below.

AIII.2 SPLICE ROUTINE

This routine has been discussed in Section IV and is presented in Figures AIII.1 through AIII.3.

AIII.3 STACKED DOUBLER ROUTINE

This routine applies only to an installation having one extra (stacked) doubler. No provision is made to account for the effect of slop or plasticity. The routine is presented in Figures AIII.4 through AIII.6.

AIII.4 STACKED SPLICE ROUTINE

This routine applies only to an installation having one extra (stacked) splice member. No provision is made to account for the effect of slop or plasticity. The routine is presented in Figures AIII.7 through AIII.9.

```
      C     PLASTIC   SPLICE
S.0001    461 FORMAT(1H1,1X,8HXAL(I,1),2X,8HXAL(I,2),2X,8HXAL(I,3),2X,8HXAL(I,4)
               1,2X,8HXAL(I,5),2X,8HXAL(I,6))
S.0002    460 FORMAT(1H1,1X,8HXKA(I,1),3X,8HXKA(I,2),3X,8HXKA(I,3),3X,8HXKA(I,4)
               1,3X,8HXKA(I,5),3X,8HXKA(I,6))
S.0003    462 FORMAT(//1X,4HXQI=,F7.0)
S.0004    459 FORMAT(3X,2HXL,5X,3HXDI,3X,3HXWD,3X,3HXLU,5X,3HXTS,3X,3HXWS,4X,
               X2HXS,2X,3HXNR,2X,3HXQQ)
S.0005    455 FORMAT(1X,4HXED=,F9.0)
S.0006    457 FORMAT(1X,3HXN=,F6.0)
S.0007    454 FORMAT(/1X,4HPLA=,F6.0)
S.0008    456 FORMAT(1X,4HXES=,F9.0)
S.0009    453 FORMAT(1H1,20X,6HSPLICE,1X,5HINPUT)
S.0010     11 FORMAT(F8.5,F6.3,F6.2,F8.5,F6.3,F6.2,F6.3,F4.0,F7.0)
S.0011    452 FORMAT(1X,4HCASE,1X,4HNO.=I10)
S.0012    451 FORMAT(//1X,13HCONFIGURATION,1X,4HNO.=,I10)
S.0013    279 FORMAT(//1X,37HFIRST FASTENER FAILURE AND TOTAL LOAD//)
S.0014    450 FORMAT(2I10)
S.0015    496 FORMAT(1X,3HSAY,1X,6HFELLOW,1H,,4HTHIS,1X,7HPROBLEM,1X,2HIS,1X,
               X3HTOO,1X,9HSENSITIVE,1H,,7HREGROUP,1X,9HFASTENERS)
S.0016     17 FORMAT(28X,6HSPLICE,1X,5HJOINT,1X,3HANS/)
S.0017     19 FORMAT(1X,2HXZ,2X,3HXNR,3X,3HXKA,7X,3HXPA,5X,3HXDL,6X,3HXKD
               1,6X,3HXQI,5X,3HXQB,8X,3HXKS)
S.0018     18 FORMAT(2F11.0)
S.0019     21 FORMAT(6F10.0)
S.0020     20 FORMAT(6F11.0)
S.0021     14 FORMAT(F6.0)
S.0022     10 FORMAT(8F10.4)
S.0023    897 FORMAT(F10.2)
S.0024     16 FORMAT(F4.0,F4.0,F9.0,2F8.0,F11.0,2F8.0,F11.0)
S.0025     13  FORMAT(F7.0)
S.0026          DIMENSION XKD(99),XKS(99)    ,XKDD(99),XKSS(99),XLSS(99)
S.0027          DIMENSION XL(99),XDI(99),XWD(99),XLK(99),XTS(99),XWS(99),
               1XS(99),XNR(99),XQQ(99),XLU(99),Z(99),XQK(99)
S.0028          DIMENSION XKA(99,6),XD(99),XPF(99),XB(99),XT(99),XTQ(99)
S.0029          INTEGER XST,XZP,XMC,XQ,XTI,XJM,XC,RYT,PLA
S.0030           DOUBLE PRECISION   XSD,XAS,XDS,XTDA,XR,XPA,XZA,XZB,XDLA,XDLB,XTD,
               1XQB,XBS,XRP,XDL,XAP(99),XLD(99),XBQ(99),XAL(99,6),XYZ,XP,XPR
               1,XAW(99),XA2(99) ,XSSP(99)
S.0031          READ(5,14) XKP
S.0032          NNP=XKP
S.0033          NKP=0
S.0034    950 CONTINUE
S.0035          WI=0.0
S.0036          WS=0.0
S.0037          READ(5,450) AA,AB
S.0038          NKP=NKP+1
S.0039          RYT=0
S.0040          READ(5,14) PLA
S.0041          READ(5,18) XED,XES
S.0042           READ(5,14) XN
```

Figure AIII.1. Splice Program

```
S.0043          WRITE(6,453)
S.0044          WRITE(6,451) AA
S.0045          WRITE(6,452) AB
S.0046          WRITE(6,454) PLA
S.0047          WRITE(6,455)XED
S.0048          WRITE(6,456) XES
S.0049          WRITE(6,457) XN
S.0050          XLRP=1.0
S.0051          N=XN
S.0052          DO 100 I=1,N
S.0053          XAW(I)=0.0
S.0054      100 Z(I)=1.
S.0055          READ(5,10)(XL(I),XDT(I),XWD(I),XLU(I),XTS(I),XWS(I),XS(I),XNR(I),
               XI=1,N)
S.0056          READ(5,897) (XQO(I),I=1,N)
S.0057          WRITE(6,459)
S.0058          WRITE(6,11) (XL(I),XDT(I),XWD(I),XLU(I),XTS(I),XWS(I),XS(I),
               1XNR(I),XQO(I),I=1,N)
S.0059          READ(5,13) XQP
S.0060          WRITE(6,462) XQP
S.0061          DO 195 I=1,N
S.0062          XKD(I)=XDT(I)*XWD(I)*XED/XLU(I)
S.0063          XKS(I)=XTS(I)*XWS(I)*XES/XLU(I)
S.0064           XLSS(I)=XS(I)
S.0065          XKSS(I)=XKS(I)
S.0066          XKDD(I)=XKD(I)
S.0067          XQK(I)=0.0
S.0068      195 CONTINUE
S.0069          XQI=XQP
S.0070          XTQ(N)=0.0
S.0071          GO TO 979
S.0072      970 CONTINUE
S.0073          XQI=-XTQ(I)+XTQ(I)/XYR*XQQK
S.0074          DO 1055 I=1,N
S.0075          XQO(I)=-XQK(I)
S.0076          XS(I)=XLSS(I)
S.0077     1055 CONTINUE
S.0078          PLA=0.0
S.0079          RYT=1.
S.0080      979 CONTINUE
S.0081          READ(5,20)(XKA(I,1), XKA(I,2),XKA(I,3),XKA(I,4),XKA(I,5),XKA(I,6)
               1,I=1,N)
S.0082          WRITE(6,460)
S.0083          WRITE(6,20)(XKA(I,1),XKA(I,2),XKA(I,3),XKA(I,4),XKA(I,5),
               1XKA(I,6),I=1,N)
S.0084          READ(5,21)(XAL(I,1),XAL(I,2),XAL(I,3),XAL(I,4),XAL(I,5),XAL(I,6)
               1,I=1,N)
S.0085          WRITE(6,461)
S.0086          WRITE(6,21)(XAL(I,1),XAL(I,2),XAL(I,3),XAL(I,4),XAL(I,5),
               1XAL(I,6),I=1,N)
S.0087          XZP=0
```

Figure AIII.1. Splice Program (Continued)

```
      XY=0
      XP=0.D0
      XAM=1.
      XII=-1.
      XST=0
      XPR=0
      XTP=0
      J=1
      I=1
      GO TO 430
  400 CONTINUE
      WT=0.0
      WS=0.0
      IF(.9999-XP) 302,302,1798
 1798 CONTINUE
      IF(XP) 401,1302,401
 1302 CONTINUE
      IF(ABS(XQI)-ABS(XQP)) 401,302,401
  401 CONTINUE
      XQI=XQI*(1.-XP)
      DO 1005 I=1,N
      XQO(I)=XQO(I)*(1.-XP)
 1005 CONTINUE
  458 CONTINUE
      XZP=0
      XY=0
      XAM=1.
      XII=-1.
      XSI=-1.
      IF(XUI) 370,430,371
  371 CONTINUE
      JJJ=ZK
      IF(Z(III)-6.) 840,840,998
  840 CONTINUE
      IKA=XAL(III,JJJ+1)
      IF(IKA) 999,995,368
  368 CONTINUE
      XKA(III,J  )=XKA(III,JJJ+1)
      XAL(III,J  )=XAL(III,JJJ+1)
      Z(III)=ZK+1.
      GO TO 370
  995 II=III
      GO TO 998
  370 CONTINUE
      JJ=YK
      Z(II)=YK+1.
      IF(Z(II)-6.) 79,79,998
   79 CONTINUE
      IKS=XAL(II,JJ+1)
      IF(IKS) 999,998,429
  429 CONTINUE
      XAL(II,J )=XAL(II,JJ+1)
```

Figure AIII.1. Splice Program (Continued)

```
      XKA(II,J )=XKA(II,JJ+1)
  430 CONTINUE
      I=1
      XAED=XDT(I)*XWD(I)*XED
      XAES=XTS(I)*XWS(I)*XES
      XPA=((8./XN)/(XAED+XAES))*XCI*XAED
      GO TO 56
   49 IF(XZP) 183,180,181
  181 XAM=.1
      XJM=1.
      XII=1.
      XPA=XR+XAM
      GO TO 32
  180 XAM=125.
      XPA=XR+XAM
      XII=0
      GO TO 32
  183 IF(XMC) 186,185,184
  184 XAM=.001
       XPA = XR+XAM
      XJM=0
      GO TO 32
  185 XAM=.00001
      XPA=XR+XAM
      XJM=-1.
      XQ=-1
      GO TO 32
  186 IF(XC) 187,188,189
  187 XAM=.0000001
      XPA=XR+XAM
      XQ=0
      GO TO 32
  188 XAM=.000000001
      XPA=XR+XAM
      XQ=1.
      GO TO 32
  189 CONTINUE
      WRITE(6,496)
      GO TO 999
   51  IF(XII) 31,34,33
   34 XAM=-5.
      XPA=XR+XAM
      XZP=1.
      GO TO 32
   33 IF(XJM) 37,36,35
   35 XAM=-.01
      XPA=XR+XAM
      XMC=1.
      XZP=-1.
      GO TO 32
   36 XAM=-.0001
```

Figure AIII.1. Splice Program (Continued)

```
      XPA=XR+XAM
      XMC=0
      GO TO 32
   37 IF(XQ) 38,39,40
   38 XAM=-.000001
      XPA=XR+XAM
      XQ=-1.
      XMC=-1.
      GO TO 32
   39 XAM=-.00000001
      XPA=XR+XAM
      XQ=0
      GO TO 32
   40 XAM=-.0000000001
      XPA=XR+XAM
      XQ=1
      GO TO 32
   31 XAM=-500.
      XPA=XR+XAM
   32 XR=XPA
      I=1
   56 XZA=XNR(I)*XPA/XKA(I,J)
      XDS=0
      XDLA=0
      XZ=0
      XQS=0
      XR=XPA
      XTDA=XR
      XTD=XZA
      GO TO 80
   81 CONTINUE
      I=I+1
      XTD=XTD-XDS
   80 CONTINUE
      XAS=XTD
      IF(XS(I+1)) 424,428,424
  424 CONTINUE
      XSD=0.0
      XPA=0.0
      IF(XLRP) 165,165,1001
 1001 CONTINUE
      IF(XZ-XN+1.) 561,165,165
  561 CONTINUE
      XKDD(I)=XKD(I)
      XKSS(I)=XKS(I)
      GO TO 165
  428 CONTINUE
      IF(I-1) 999,426,425
  425 CONTINUE
      IF(XS(I)) 427,426,427
  427 CONTINUE
      XPA=0.0
```

Figure AIII.1. Splice Program (Continued)

```
      XKDD(I)=XKD(I)
      XKSS(I)=XKS(I)
  426 CONTINUE
      XPA=XAS*XKA(I,J)
  165 CONTINUE
      XDLA=XDLA+XPA*XNR(I)
      XSD=XLDA/XKDD(I)
      XQS=XL(I)*XCC(I)+XQS
      XQT=XQS+XQI
      XQB=XQT-XDLA
      XBS=XQB/XKSS(I)
      XDS=XBS-XSD
      XZ=XZ+1.
      IF(XST) 589,598,589
  598 XYR=XQS+XQP
      XQCK=XQS
  589 CONTINUE
   83 IF(XN-XZ) 71,71,81
   88 CONTINUE
      IF(DABS(XLD(I)-XQT)-1.) 72,72,857
  857 CONTINUE
      XZA=XTDA
      XDLA=XLD(I)
      XR=XRP*XKA(I,J)
      I=1
      XZB=XZA+(XDLA-XQTA)*(XZA-XZB)/(XDLB -XQTB-XDLA+XQTA)
      XPA=XKA(I,J)*(XZB)
       IF(XZB-XZA) 95,999,95
   71 I=1
      XQTA=XQT
      IF(XQT) 233,999,238
  233 IF(XDLA) 2005,2005,2006
 2006 GO TO 51
 2005 IF(XDLA/XQT - .75) 51,53,2007
 2007 IF(XDLA/XQT - 1.0) 53,53,49
  238 IF(XDLA) 2008,2008,2009
 2008 GO TO 49
 2009 IF(XDLA/XQT - .75) 49,53,2010
 2010 IF(XDLA/XQT - 1.0) 53,53,51
   53 CONTINUE
      XPA=XR+XAM/10.
  124 XZB=XNR(I)*XPA/XKA(I,J)
   95 XTD=XZB
      XR=XPA
      XDS=0
      XDLB=0
      XZ=0
      XQS=0
      GO TO 84
   85 CONTINUE
      I=I+1
   84 XTD=XTD-XDS
```

Figure AIII.1. Splice Program (Continued)

```
      XAS=XTD
      IF(XS(I)) 419,418,419
  419 CONTINUE
      XSSP(I)=XTD
      XSD=0.0
      XPA=0.0
      GO TO 265
  418 CONTINUE
      XPA=XAS*XKA(I,J)
  265 CONTINUE
      XDLB=XDLB+XPA*XNR(I)
      XSD=XDLB/XKDD(I)
      XQS=XL(I)*XQO(I)+XQS
      XQT=XQS+XQI
      XQB=XQT-XDLB
      XBS=XQB/XKSS(I)
      XDS=XBS-XSD
      XZ=XZ+1.
   87 CONTINUE
      IF(XN-XZ) 104,104,85
  104 CONTINUE
      XQTB=XQT
      XPR=0
      XZ=0
      I=1
      XLD(I)=0
      XQS=0
      XDS=0
      XY=0
      XPI=0
      XVF=XP
      XUT=0
      XP=0.0
  131 XTD=XZB+(XDLB-XQTB)*(XZB-XZA)/(XDLA-XQTA-XDLB+XQTB)
      XTDA=XTD
  132 XRP=XTDA
      GO TO 86
   74 CONTINUE
      I=I+1
      XTD=XTD-XDS
   86 CONTINUE
      XAS=XTD
      IF(XS(I)) 409,408,409
  409 CONTINUE
      XAP(I)=0.0
      XSSP(I)=XTD
      WT=(DABS(XTD)-XS(I))/DABS(XTD)
      IF(WT) 389,390,390
  389 CONTINUE
      WT=0.0
      GO TO 332
```

Figure AIII.1. Splice Program (Continued)

```
  390 CONTINUE
      WT=ABS(WT)
      IF(WT-XP) 332,374,375
  375 XP=WT
      II=I
      GO TO 332
  374 CONTINUE
      III=I
      GO TO 332
  408 CONTINUE
      XAP(I)=XAS*XKA(I,J)
      XA2(I)=XTD
      IF(RYT) 648,648,331
  648 CONTINUE
      IF(XST) 937,909,999
  909 CONTINUE
      XPE(I)=0
  937 CONTINUE
      XYZ=XAL(I,J)-ABS(XPE(I))
      IF(DABS(XYZ)-DABS(XAP(I))) 306,306,331
  306 WT=DABS(XYZ/XAP(I))
      WS=XP
      WT=1.-WT
      IF( WT-WS) 331,309,308
  309 CONTINUE
      III=I
      ZK=Z(I)
      XUT=1.
      GO TO 332
  308 II=I
      YK=Z(I)
      XUT=-1.
      XPI=1.
      XP=DABS(XYZ/ XAP(I))
      XP=1.-XP
      GO TO 332
  331 CONTINUE
  332 IF(I-1) 750,775,750
  775 XLD(I)=XAP(I)*XNR(I)
      GO TO 800
  750 XLD(I)=XLD(I-1)+XAP(I)*XNR(I)
  800 CONTINUE
      XSD=XLD(I)/XKDD(I)
      XQS=XL(I)*XQ2(I)+XQS
      XQT=XQS+XQT
      XBQ(I)=XQT-XLD(I)
      XBS=XBQ(I)/XKSS(I)
      XDS=XBS-XSD
      XZ=XZ+1.
  420 IF(XN-XZ) 102,102,74
  102 CONTINUE
  421 XQTA=XQT
```

Figure AIII.1. Splice Program (Continued)

```
      IF(DABS(XLD(I) -XOT)-DABS(.01*XAP(I))) 72,72,88
   72 CONTINUE
      IF(XS(II)*1000.) 481,482,481
  481 CONTINUE
      XSSP(II)=0.0
      XLT=0.0
      XS(II)=0
      XKDD(II)=XKD(II)
      XKSS(II)=XKS(II)
      IF(II-1) 479,482,479
  479 CONTINUE
      XKDD(II-1)=XKD(II-1)
      XKSS(II-1)=XKS(II-1)
  482 CONTINUE
      IF(XS(III)*1000.)  515,422,515
  515 CONTINUE
      XS(III)=0.0
      XSSP(III)=0.0
      XKDD(III)=XKD(III)
      XKSS(III)=XKS(III)
      XKDD(III-1)=XKD(III-1)
      XKSS(III-1)=XKS(III-1)
  422 CONTINUE
      XP=1.-XP
      DO 1000 I=1,N
      XS(I)=XS(I)-DABS(XSSP(I)*XP)
 1000 CONTINUE
      IF(RYT) 70,70,359
   70 CONTINUE
      IF(XP) 359,300,359
  300 XP=1.
  359 CONTINUE
      I=1
      XZ=1.
      IF(XST) 737,707,999
  707 CONTINUE
      IF(RYT) 708,708,737
  708 CONTINUE
      GO TO 736
  735 I=I+1
  736 CONTINUE
      X9(I)=0
      XD(I)=0
      XTQ(I)=0
      XPF(I)=0
      IF(N-I) 999,734,735
  734 I=1
      GO TO 737
   65 CONTINUE
      I=I+1
      XZ=XZ+1.
  737 CONTINUE
```

Figure AIII.1. Splice Program (Continued)

```
      XQK(I) =XQC(I)*XP + XQK(I)
      XPF(I)=XP*XAP(I)+XPF(I)
      XD(I)=XLD(I)*XP+XD(I)
      XB(I)=XBC(I)*XP+XB(I)
      XTQ(I)=(XBQ(I)+XLD(I))*XP+XTQ(I)
      XBQ(I)=XTQ(I)-XD(I)
      XLRP=0.0
      IF(XN-XZ) 301,301,65
  301 CONTINUE
      IF(RYT) 485,485,486
  486 CONTINUE
      ITQ=XTQ(I)
      IF(ITQ) 400,302,400
  485 CONTINUE
      IYR=XYR
      ITQ=XTQ(I)
  711 CONTINUE
      IF(IYR-ITQ) 505,302,400
  505 ABC=IABS(IYR-ITQ)
      IF(ABC-.0001*XYR) 302,302,305
  302 I=1
      GO TO 304
  998 CONTINUE
      XI=II
      WRITE(6,279)
      WRITE(6,18) XI,XTQ(I)
      GO TO 302
  303 I=I+1
      XZ=XZ+1.
      GO TO 410
  304  WRITE(6,17)
       WRITE(6,19)
      XZ=1.
  410 CONTINUE
      WRITE(6,16) XZ,XNR(I),XKA(I,J),XPF(I),XD(I),XKD(I),XTQ(
     1XB(I),XKS(I)
      IF(XN-XZ) 999,999,303
  305 XP=(XYR-XTQ(I))/XYR
      XZ=1.
      I=1
      GO TO 737
  999 CONTINUE
      IF(PLA) 980,980,970
  980 CONTINUE
      IF(NKP-NNP) 950,951,951
  951 CONTINUE
      STOP
      END
```

Figure AIII.1. Splice Program (Concluded)

```
1.
1        3
1.
10300000.   10300000.
20.
1.0    .072   1.38   1.0   .102   2.88   .1     1.0
1.0    .072   1.38   1.0   .102   2.88   .1     1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .0     1.0
1.0    .072   1.38   1.0   .102   2.88   .0     1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
1.0    .072   1.38   1.0   .102   2.88   .001   1.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
18000.
117500.   105600.   69700.   32000.   19200.   12900.
117500.   105600.   69700.   32000.   19200.   12900.
117500.   105600.   69700.   32000.   19200.   12900.
117500.   105600.   69700.   32000.   19200.   12900.
117500.   105600.   69700.   32000.   19200.   12900.
```

Figure AIII.2 Splice Program Input Data

117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
117500.					
117500.					
117500.					
117500.					
117500.					
117500.					
117500.					
117500.					
117500.					
117500.					
117500.					
117500.					
117500.					
117500.					
117500.					
117500.					

Figure AIII.2 Splice Program Input Data (Concluded)

```
      117500.
      117500.
      117500.
      117500.
 10000.
 10000.
 10000.
 10000.
 10000.
 10000.
 10000.
 10000.
 10000.
 10000.
 10000.
      10000.
 10000.
 10000.
 10000.
 10000.
 10000.
 10000.
 10000.
 10000.
```

Figure AIII.3 Splice Program Output Data

SPLICE INPUT

CONFIGURATION NO.= 1000000
CASE NO.= 3000000

PLA= 1.
XED=10300000.
XES=10500000.
XN= 20.

XL	XCT	XWL	XLU	XTS	XKS	XS	XNR	XQO
1.00000	0.072	1.38	1.00000	0.102	2.88	0.100	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.100	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.
1.00000	0.072	1.38	1.00000	0.102	2.88	0.001	1.	0.

XQI= 18000.

XKA(I,1)	XKA(I,2)	XKA(I,3)	XKA(I,4)	XKA(I,5)	XKA(I,6)
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.

Figure AIII.3 Splice Program Output Data (Continued)

117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.
117500.	105600.	69700.	32000.	19200.	12900.

XAL(I,1)	XAL(I,2)	XAL(I,3)	XAL(I,4)	XAL(I,5)	XAL(I,6)
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.
750.	1125.	1390.	1550.	1670.	1750.

FIRST FASTENER FAILURE AND TOTAL LOAD

20. 11422.

SPLICE JOINT ANS

XI	ANK	XKA	XPA	XDL	XKD	XQT	XQB	XKS
1.	1.	117500.	0.	0.	1023408.	11422.	11422.	3025728.
2.	1.	117500.	0.	0.	1023408.	11422.	11422.	3025728.
3.	1.	105600.	934.	934.	1023408.	11422.	10487.	3025728.
4.	1.	117500.	649.	1583.	1023408.	11422.	9839.	3025728.
5.	1.	117500.	441.	2024.	1023408.	11422.	9398.	3025728.
6.	1.	117500.	300.	2324.	1023408.	11422.	9097.	3025728.
7.	1.	117500.	204.	2528.	1023408.	11422.	8894.	3025728.
8.	1.	117500.	137.	2665.	1023408.	11422.	8757.	3025728.
9.	1.	117500.	204.	2869.	1023408.	11422.	8552.	3025728.
10.	1.	117500.	202.	3071.	1023408.	11422.	8351.	3025728.
11.	1.	117500.	124.	3195.	1023408.	11422.	8226.	3025728.
12.	1.	117500.	182.	3377.	1023408.	11422.	8044.	3025728.
13.	1.	117500.	266.	3643.	1023408.	11422.	7779.	3025728.
14.	1.	117500.	390.	4033.	1023408.	11422.	7389.	3025728.
15.	1.	117500.	573.	4606.	1023408.	11422.	6816.	3025728.
16.	1.	105600.	834.	5439.	1023408.	11422.	5982.	3025728.
17.	1.	69700.	1169.	6608.	1023408.	11422.	4814.	3025728.

Figure AIII.3 Splice Program Output Data (Continued)

18.	1.	22000.	1446.	8054.	1023408.	11422.	3368.	3025728.
19.	1.	19200.	1618.	9672.	1023408.	11422.	1750.	3025728.
20.	1.	12900.	1750.	11422.	1023408.	11422.	0.	3025728.

XKA(1,1)	XKA(1,2)	XKA(1,3)	XKA(1,4)	XKA(1,5)	XKA(1,6)
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.
117500.	-0.	-0.	-0.	-0.	-0.

XAL(1,1)	XAL(1,2)	XAL(1,3)	XAL(1,4)	XAL(1,5)	XAL(1,6)
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.
10000.	-0.	-0.	-0.	-0.	-0.

Figure AIII.3 Splice Program Output Data (Concluded)

SPLICE JOINT ANS								
XZ	XNK	XKA	XPA	XCL	XKD	XCT	XQB	XKS
1.	1.	117500.	0.	0.	1023408.	-0.	-0.	3025728.
2.	1.	117500.	0.	0.	1023408.	-0.	-0.	3025728.
3.	1.	117500.	2.	2.	1023408.	0.	-2.	3025728.
4.	1.	117500.	16.	18.	1023408.	0.	-18.	3025728.
5.	1.	117500.	12.	30.	1023408.	-0.	-30.	3025728.
6.	1.	117500.	8.	38.	1023408.	0.	-38.	3025728.
7.	1.	117500.	4[illegible]	41.	1023408.	-0.	-41.	3025728.
8.	1.	117500.	-2.	39.	1023408.	-0.	-39.	3025728.
9.	1.	117500.	3.	42.	1023408.	0.	-42.	3025728.
10.	1.	117500.	9.	51.	1023408.	-0.	-51.	3025728.
11.	1.	117500.	29.	80.	1023408.	-0.	-80.	3025728.
12.	1.	117500.	51.	131.	1023408.	0.	-131.	3025728.
13.	1.	117500.	80.	210.	1023408.	-0.	-210.	3025728.
14.	1.	117500.	120.	330.	1023408.	0.	-330.	3025728.
15.	1.	117500.	177.	507.	1023408.	0.	-507.	3025728.
16.	1.	117500.	252.	759.	1023408.	-0.	-759.	3025728.
17.	1.	117500.	312.	1071.	1023408.	-0.	-1071.	3025728.
18.	1.	117500.	181.	1252.	1023408.	-0.	-1252.	3025728.
19.	1.	117500.	-248.	1004.	1023408.	0.	-1004.	3025728.
20.	1.	117500.	-1004.	0.	1023408.	-0.	-0.	3025728.

Figure AIII.4. Stacked Doubler Program

```
C     STACKED DOUBLERS
  465 FORMAT(/3X,2HXL,8X,3HXKA,7X,4HXKD1,7X,4HXKD2,9X,3HXKS,6X,2HXS,
     17X,3HXNR,4X,3HXCC)
  463 FORMAT(1X,5HXAED=,F9.0)
  464 FORMAT(1X,5HXAES=,F9.0,
  462 FORMAT(//1X,4HXCI=,F7.0)
  457 FORMAT(1X,3HXN=,F6.0)
  453 FORMAT(1H1,20X,7HDOUBLER,1X,5HINPUT)
  451 FORMAT(//1X,13HCONFIGURATION,1X,4HNO.=,I10)
  452 FORMAT(1X,4HCASE,1X,4HNO.=I10)
  450 FORMAT(2I10)
  456 FORMAT(1X,3HSAY,1X,6HFELLOW,1H,,4HTHIS,1X,7HPROBLEM,1X,2HIS,1X,
     X3HTOO,1X,9HSENSITIVE,1H,,7HREGROUP,1X,9HFASTENERS)
   17 FORMAT(34X,7HDOUBLER,1X,3HANS/)
   19 FORMAT(4X,2HXZ,5X,3HXQT,7X,4HXAP1,5X,3HXL1,6X,4HXAP2,6X,3HXD2,7X,
     13HXBS)
   18 FORMAT(2F11.0)
   14 FORMAT(F6.0)
   10 FORMAT(F10.5,4F10.0,F10.3,2F10.0)
   13 FORMAT(F7.0)
   16 FORMAT(F7.0,2F9.0,F10.0,F9.0,2F10.0)
      DIMENSION XL(99), XKA(99), XKS(99),   XS(99), XNR(99), XQO(99)
     1,XKD(99,2),XCK(99),XJA(99)
       DOUBLE PRECISION   XSD,XAS,XDS,XTCA,XB,XPA,XZA,XZB,XDLA,XDLB,XTD,
     1XCB,XES,XRP,XLL,PXA(99),SDX(99),XSB(99),TP(99),XAP(99),SXD(99)
      NNP=XKP
      NKP=0
  950 CONTINUE
      NKP=NKP+1
  500 CONTINUE
      XAA=0
      READ(5,450) AA,AB
      WRITE(6,453)
      WRITE(6,451) AA
      WRITE(6,452) AB
      READ(5,14) XKP
      READ(5,14) XNN
       READ(5,18) XAED,XAES
      WRITE(6,463) XAED
      WRITE(6,464) XAES
       READ(5,14) XN
      WRITE(6,457) XN
       READ(5,13) XQI
      WRITE(6,462) XQI
      XAM=1.
      N=XN
      XZP=0
      XTT=-1.
      XY=0
      READ(5,10) (XL(I),XKA(I),XKD(I,1),XKD(I,2),XKS(I),XS(I),XNR(I),
     1XQO(I),I=1,N)
```

Figure AIII.4. Stacked Doubler Program (Continued)

```
      WRITE(6,405)
      WRITE(6,10)(XL(I),XKA(I),XKD(I,1),XKD(I,2),XKS(I),XS(I),XNR(I),
     1XQ(I),I=1,N)
      DO 1000 I=1,N
 1000 XDR(I)=XKD(I,1)+XKD(I,2)
      XPA=((8./XN)/(XAEC+XAES))*XL1*XAEC
      I=1
      GO TO 50
   49 IF(XZP) 183,180,181
  181 XAM=.1
      XJM=1.
      XII=1.
      XPA=XR+XAM
      GO TO 32
  180 XAM=100.
      XPA=XR+XAM
      XII=0
      GO TO 32
  183 IF(XJM) 186,185,184
  184 XAM=.001
       XPA = XR+XAM
      XJM=0
      GO TO 32
  185 XAM=.00001
      XPA=XR+XAM
      XJM=-1.
      XL=-1
      GO TO 32
  186 IF(XL) 187,188,189
  187 XAM=.0000001
      XPA=XR+XAM
      XL=0
      GO TO 32
  188 XAM=.000000001
      XPA=XR+XAM
      XL=1.
      GO TO 32
  189 CONTINUE
      WRITE(6,496)
      GO TO 999
   51 IF(XII) 31,34,33
   34 XAM=-5.
      XPA=XR+XAM
      XZP=1.
      GO TO 32
   33 IF(XJM) 37,36,35
   35 XAM=-.01
      XPA=XR+XAM
      XML=1.
      XZP=-1.
      GO TO 32
```

Figure AIII.4. Stacked Doubler Program (Continued)

```
   36 XAM=-.0001
      XPA=XR+XAM
      XMC=0
      GO TO 32
   37 IF(XC) 38,39,40
   38 XAM=-.000001
      XPA=XR+XAM
      XO=-.1
      XMC=-1.
      GO TO 32
   39 XAM=-.00000001
      XPA=XR+XAM
      XO=0
      GO TO 32
   40 XAM=-.0000000001
      XPA=XR+XAM
      XO=1
      GO TO 32
   31 XAM=-500.
      XPA=XR+XAM
   32 XR=XPA
      I=1
      XZ=0
      XRZ=1.
      IF(XIV) 204,56,56
   56 XZA=XNR(I)*XPA/XKA(I)+XS(I)
      XDS=0
      XDLA=0
      XRZ=0
      XZ=0
      XCS=0
      XR=XPA
      XIDA=XR
      XID=XZA
      GO TO 80
   81 CONTINUE
      I=I+1
      XID=XID-XDS
   80 XAS=XID-XS(I)
      XZ=XZ+1.
      XPA=XAS*XKA(I)
      XDLA=XDLA+XPA*XNR(I)
      XSD=XDLA/XDK(I)
      XQS=XL(I)*XCC(I)+XCS
      XQT=XQS+XQI
      XXI=XCT
      XCB=XQT-XDLA
      XBS=XCB/XKS(I)
      XCS=XBS-XSD
      IF(XCT) 233,999,2400
  233 CONTINUE
```

Figure AIII.4. Stacked Doubler Program (Continued)

```
      IF(XDLA/XGT-3.) 42,42,49
   42 IF(3.-XDLA/XGT) 51,53,53
 2400 CONTINUE
      IF(XDLA-3.*XGT) 57,57,51
   57 IF(3.*XGT+XDLA) 49,53,53
   53 IF(XN-XZ) 101,101,81
  101 CONTINUE
      IF((DABS(XDLA))-.01*DABS(XPA)) 70,70,83
   83 CONTINUE
      GO TO 71
   88 CONTINUE
      XZA=XTDA
      XDLA=XDL
      XR=XNP*XKA(I)
      I=1
      XZB=XZA+XDLA*(XZA-XZB)/(XDLB-XDLA)
      XPA=XKA(I)*(XZB-XS(I))
       IF(XZB-XZA) 95,999,95
   71 I=1
      XPA=XR+XAM
  124 XZB=XNR(I)*XPA/XKA(I)+XS(I)
   95 XTD=XZB
      XR=XPA
      XDS=0
      XII=-1.
      XZP=0
      XDLB=0
      XZ=0
      XGS=0
      GO TO 84
   85 CONTINUE
      I=I+1
   84 XTD=XTD-XDS
      XAS=XTD-XS(I)
      XPA=XAS*XKA(I)
      XZ=XZ+1.
      XDLB=XDLB+XPA*XNR(I)
      XSD=XDLB/XDK(I)
      XGS=XL(I)*XGC(I)+XGS
      XGT=XGS+XGI
      XGB=XGT-XGLF
      XBS=XGB/XKS(I)
      XDS=XBS-XSD
      IF(XN-XZ) 103,103,85
  103 CONTINUE
      IF((DABS(XDLB))-.01*DABS(XPA)) 70,70,87
   87 CONTINUE
      XZ=0
      XDL=0
      XGS=0
      XDS=0
      XY=0
```

Figure AIII.4. Stacked Doubler Program (Continued)

```
      I=1
  131 XTD=XZB+XDLE*(XZB-XZA)/(XDLA-XDLE)
      XTDA=XTD
  132 XRP=XTDA
      XR=XPA
      GO TO 66
   74 CONTINUE
      I=I+1
      XTD=XTD-XDS
   86 XAS=XTD-XS(I)
      XZ=XZ+1.
      XAP(I)=XAS*XKA(I)
      XDL=XDL+XAP(I)*XNR(I)
      SXD(I)=XDL/XDK(I)
      XQS=XL(I)*XQC(I)+XQS
      XQT=XQS+XQI
      XQB=XQT-XDL
      XBS=XQB/XKS(I)
      XDS=XBS-SXD(I)
  117 IF(XN-XZ) 102,102,74
  102 CONTINUE
      IF((DABS(XDL ))-.01*DABS(XPA)) 70,70,88
   70 CONTINUE
      XTS=0
      I=1
      XQT=0
      XST=XAP(I)*XNR(I)
      XZ=0
  204 CONTINUE
      XTV=-1.
      NT=I
      XQT=XST
      IF(XRZ) 240,318,240
  318 CONTINUE
      XPA=XKD(1,1)/(XKD(1,1)+XKD(1,2))*XST
  240 CONTINUE
      XTD=0
      XDLA=0
      XDS=0
      XR=XPA
      IF(XKA(I)/XKD(I,2)-100.) 337,4338,4338
 4338 XJA(I)=XKD(I,2)
      GO TO 336
  337 XJA(I)=XKA(I)
  336 CONTINUE
      XZA=XNR(I)*XPA/XJA(I)+XS(I)
      XTDA=XR
      XTD=XZA
      GO TO 202
  201 CONTINUE
      XTD=XTD-XDS
```

Figure AIII.4. Stacked Doubler Program (Continued)

```
      I=I+1
      XCT=XAP(I)*XNR(I)+XCT
  202 CONTINUE
      XAS=XTD-XS(I)
      XZ=XZ+1.
      IF(XKA(I)/XKD(I,2)-100.) 339,335,335
  335 XJA(I)=XKD(I,2)
      IF(XN-XZ) 339,339,338
  339 XJA(I)=XKA(I)
  338 CONTINUE
      XPA=XAS*XJA(I)
      XDLA=XDLA+XPA*XNR(I)
      XSC=XDLA/XKD(I,2)
      XQB=XCT-XDLA
      XSB(I)=XQB/XKD(I,1)
      XDS=XSB(I)-XSD
  208 CONTINUE
       IF( XXT ) 333,999,340
  333 CONTINUE
      IF(XDLA/XXT-3.) 142,142,49
  142 IF(3.+XDLA/XXT) 51,239,239
  340 CONTINUE
      IF(XDLA-3.*XXT) 238,238,51
  238 CONTINUE
      IF(3.*XXT+XDLA) 49,239,239
  239 CONTINUE
      IF(XN-XZ) 203,203,201
  203 CONTINUE
      I=1
      XTS=0
      XPA=XR+XAM
      XZ=0
      XST=XAP(I)*XNR(I)
  206 XZB=XNR(I)*XPA/XJA(I)+XS(I)
      XTD=XZB
      XR=XPA
      XDLB=0
      XZ=0
      XCS=0
      XDS=0
      XQT=XST
      GO TO 210
  211 CONTINUE
      I=I+1
      XTD=XTD-XDS
      XQT=XAP(I)*XNR(I)+XQT
  210 XAS=XTD-XS(I)
      XPA=XAS*XJA(I)
      XZ=XZ+1.
      XDLB=XDLB+XPA*XNR(I)
      XSD=XDLB/XKD(I,2)
```

Figure AIII.4. Stacked Doubler Program (Continued)

```
      XQB=XQT-XDLB
      XSB(I)=XQB/XKD(I,1)
      XDS=XSB(I)-XSD
      IF(XN-XZ) 212,212,211
  212 CONTINUE
      XZ=0
      XDL=C
      XQS=C
      XDS=C
      XY=0
      I=1
      XTS=C
      XST=XAP(I)*XNR(I)
      XQT=XST
      XTD=XZB+XDLB*(XZB-XZA)/(XDLA-XDLB)
      XTDA=XTD
      XRP=XTDA
      GO TC 221
  22C CONTINUE
      I=I+1
      XTD=XTD-XDS
      XQT=XAP(I)*XNR(I)+XQT
  221 XAS=XTD-XS(I)
      XZ=XZ+1.
      PXA(I)=XAS*XJA(I)
      XDL=XDL+PXA(I)*XNR(I)
      SDX(I)=XDL/XKD(I,2)
      XQB=XQT-XDL
      XSB(I)=XQB/XKD(I,1)
      XDS=XSB(I)-SDX(I)
      TP(I)=SXD(I)/XSB(I)
      XDK(I)=TP(I)*XCK(I)
      IF(XN-XZ) 222,222,220
  222 CONTINUE
      DO 1001 I=1,N
      PXX=PXA(NT)
      IF(XAA-PXX) 10C2,1001,1002
 1001 CONTINUE
      GC TC 250
 10C2 XPA=TP(NT)*XAP(NT)
      I=1
      XAA=PXA(NT)
      XZ=0
      XZP=C
      XTT=-1.
      XTV= 1.
      GC TC 56
  250 I=1
      XZ=0
      WRITE(6,17)
      WRITE(6,19)
```

Figure AIII.4. Stacked Doubler Program (Concluded)

```
      XD1=C
      XD2=0
      GC TC 252
  251 CCNTINUE
      I=I+1
  252 CONTINUE
      XZ=XZ+1.
      XCT=XD1+XL(I)*XGC(I)
      XD1=XD1+XAP(I)*XNR(I)-PXA(I)*XNR(I)
      XD2=XD2+PXA(I)*XNR(I)
      XBS=XDT-XD1-XD2
      WRITE(6,16) XZ,XCT,XAP(I),XD1,PXA(I),XD2,XBS
      IF(XN-XZ) 999,999,251
  999 CCNTINUE
      IF(NKF-NNP) 95C,951,951
  951 CONTINUE
      STOP
      END
```

Figure AIII.5. Stacked Doubler Program Input Data

1	1						
1.							
17.							
1030000.	1030000.						
20.							
12400.							
1.0	117000.	1030000.	1000.	3190000.	.0	1.	0.
1.0	117000.	1030000.	1000.	3190000.	.0	1.	0.
1.0	117000.	1030000.	1000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1030000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1030000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1030000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1030000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1030000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1030000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1030000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1030000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1030000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1030000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1030000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1030000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1030000.	3190000.	.0	1.	0.
1.0	259000.	1030000.	1000.	3190000.	.0	1.	0.
1.0	117000.	1030000.	1000.	3190000.	.0	1.	0.
1.0	117000.	1030000.	1000.	3190000.	.0	1.	0.
1.0	117000.	1030000.	1000.	3190000.	.0	1.	0.

Figure AIII.6. Stacked Doubler Program Output Data

DOUBLER INPUT

CONFIGURATION NO.= 1000000
CASE NO.= 1000000
XAEC= 1030000.
XAES= 1030000.
XN= 20.

XQI= 12400.

XL	XAA	XKD1	XKD2	XKS	XS	XNR	XCO
1.00000	117000.	1030000.	1000.	3190000.	0.	1.	0.
1.00000	117000.	1030000.	1000.	3190000.	0.	1.	0.
1.00000	117000.	1030000.	1000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1030000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1030000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1030000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1030000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1030000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1030000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1030000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1030000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1030000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1030000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1030000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1030000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1030000.	3190000.	0.	1.	0.
1.00000	259000.	1030000.	1000.	3190000.	0.	1.	0.
1.00000	117000.	1030000.	1000.	3190000.	0.	1.	0.
1.00000	117000.	1030000.	1000.	3190000.	0.	1.	0.
1.00000	117000.	1030000.	1000.	3190000.	0.	1.	0.

DOUBLER ANS

XZ	XCT	XAP1	XC1	XAP2	XC2	XBS
1.	12400.	1658.	1856.	2.	2.	10542.
2.	12400.	1471.	3325.	2.	4.	9071.
3.	12400.	1139.	4460.	4.	8.	7932.
4.	12400.	1877.	4485.	1852.	1860.	6055.
5.	12400.	1385.	4679.	1191.	3051.	4670.
6.	12400.	[illegible]	4903.	782.	3833.	3664.
7.	12400.	709.	5099.	513.	4346.	2955.
8.	12400.	469.	5244.	323.	4669.	2486.
9.	12400.	267.	5232.	179.	4848.	2220.
10.	12400.	87.	5261.	57.	4906.	2133.
11.	12400.	-87.	5332.	-57.	4848.	2220.
12.	12400.	-267.	5244.	-179.	4669.	2486.
13.	12400.	-469.	5099.	-323.	4346.	2955.
14.	12400.	-709.	4903.	-513.	3833.	3664.
15.	12400.	-1006.	4678.	-782.	3052.	4670.
16.	12400.	-1305.	4484.	-1191.	1861.	6055.

Figure AIII.7. Stacked Splice Program

17.	12400.	-1877.	4457.	-1850.	11.	7932.
18.	12400.	-1139.	3326.	-8.	3.	9071.
19.	12400.	-1471.	1856.	-1.	2.	10542.
20.	12400.	-1858.	-0.	-2.	0.	12400.

Figure AIII.7. Stacked Splice Program (Continued)

```
C     STACKED SPLICES
   14 FORMAT(F6.0)
  465 FORMAT(/3X,2HXL,8X,3HXKA,7X,4HXKD1,7X,4HXKD2,9X,3HXKS,6X,2HXS,
     17X,3HXNR,4X,3HXQU)
  464 FORMAT(1X,5HXAES=,F9.0)
  463 FORMAT(1X,5HXAED=,F9.0)
  462 FORMAT(//1X,4HXQI=,F7.0)
  457 FORMAT(1X,3HXN=,F6.0)
  453 FORMAT(1H1,20X,6HSPLICE,1X,5HINPUT)
  450 FORMAT(2I10)
  452 FORMAT(1X,4HCASE,1X,4HNO.=I10)
  451 FORMAT(//1X,13HCONFIGURATION,1X,4HNO.=,I10)
  496 FORMAT(1X,3HSAY,1X,6HFELLOW,1H,,4HTHIS,1X,7HPROBLEM,1X,2HIS,1X,
     X3HTOO,1X,9HSENSITIVE,1H,,7HREGROUP,1X,9HFASTENERS)
   17 FORMAT(28X,6HSPLICE,1X,5HJOINT,1X,3HANS//)
   19 FORMAT(4X,2HXZ,5X,3HXQT,7X,4HXAP1,5X,3HXD1,6X,4HXAP2,6X,3HXD2,7X,
     13HXBS)
   18 FORMAT(2F11.0)
   10 FORMAT(F10.5,4F10.0,F10.3,2F10.0)
   13  FORMAT(F7.0)
   16 FORMAT(F7.0,2F9.0,F10.0,F9.0,2F10.0)
      DIMENSION XL(99), XKA(99), XKS(99),   XS(99), XNR(99), XQU(99)
     1,XKD(99,2),XCK(99),XJA(99)
       DOUBLE PRECISION   XSD,XAS,XDS,XTDA,XR,XPA,XZA,XZB,XDLA,XDLB,XTD,
     1XCB,XBS,XRP,XDL,PXA(99),SDX(99),XSB(99),TP(99),XAP(99),SXD(99)
      READ(5,14) XKP
      NNP=XKP
      NKP=0
      NKP=NKP+1
  550 CONTINUE
  500 CONTINUE
      XAA=0
      READ(5,450) AA,AB
      READ(5,14) XNN
       READ(5,18) XAED,XAES
       READ(5,14) XN
       READ(5,13) XQI
      WRITE(6,451) AA
      WRITE(6,452) AB
      WRITE(6,453)
      WRITE(6,457) XN
      WRITE(6,462) XQI
      WRITE(6,463) XAED
      WRITE(6,464) XAES
      XAM=1.
      N=XN
      XZP=0
      XTT=-1.
      XY=0
      READ(5,10) (XL(I),XKA(I),XKD(I,1),XKD(I,2),XKS(I),XS(I),XNR(I),
     1XQU(I),I=1,N)
```

Figure AIII.7. Stacked Splice Program (Continued)

```
      WRITE(6,465)
      WRITE(6,10)(XL(I),XKA(I),XKD(I,1),XKD(I,2),XKS(I),XS(I),XNR(I),
     1XQU(I),I=1,N)
      DO 1000 I=1,N
 1000 XDK(I)=XKD(I,1)+XKD(I,2)
      XPA=((8./XN)/(XAED+XAES))*XQ1*XAED
      I=1
      GO TO 56
   49 IF(XZP) 183,180,181
  181 XAM=.1
      XJM=1.
      XTT=1.
      XPA=XR+XAM
      GO TO 32
  180 XAM=125.
      XPA=XR+XAM
      XTT=0
      GO TO 32
  183 IF(XMC) 186,185,184
  184 XAM=.001
       XPA = XR+XAM
      XJM=0
      GO TO 32
  185 XAM=.00001
      XPA=XR+XAM
      XJM=-1.
      XQ=-1
      GO TO 32
  186 IF(XQ) 187,188,189
  187 XAM=.0000001
      XPA=XR+XAM
      XQ=0
      GO TO 32
  188 XAM=.000000001
      XPA=XR+XAM
      XQ=1.
      GO TO 32
  189 CONTINUE
      WRITE(6,496)
      GO TO 999
   51  IF(XTT) 31,34,33
   34 XAM=-5.
      XPA=XR+XAM
      XZP=1.
      GO TO 32
   33 IF(XJM) 37,36,35
   35 XAM=-.01
      XPA=XR+XAM
      XMC=1.
      XZP=-1.
      GO TO 32
```

Figure AIII.7. Stacked Splice Program (Continued)

```
   36 XAM=-.0001
      XPA=XR+XAM
      XMC=0
      GO TO 32
   37 IF(XC) 38,39,40
   38 XAM=-.000001
      XPA=XR+XAM
      XO=-.1
      XMC=-1.
      GO TO 32
   39 XAM=-.00000001
      XPA=XR+XAM
      XO=0
      GO TO 32
   40 XAM=-.0000000001
      XPA=XR+XAM
      XO=1
      GO TO 32
 31   XAM=-500.
      XPA=XR+XAM
   32 XR=XPA
      I=1
      XZ=0
      XRZ=1.
      IF(XTV) 204,56,56
   56 XZA=XNR(I)*XPA/XKA(I)+XS(I)
      XDS=0
      XDLA=0
      XRZ=0
      XZ=0
      XQS=0
      XR=XPA
      XTDA=XR
      XTD=XZA
      GO TO 80
   81 CONTINUE
      I=I+1
      XTD=XTD-XDS
   80 XAS=XTD-XS(I)
      XZ=XZ+1.
      XPA=XAS*XKA(I)
      XDLA=XDLA+XPA*XNR(I)
      XSD=XDLA/XDK(I)
      XQS=XL(I)*XCC(I)+XQS
      XQT=XQS+XQI
      XAT=XQI
      XQB=XQT-XDLA
      XBS=XQB/XKS(I)
      XLS=XBS-XSD
      IF(XQT) 233,999,2400
  233 CONTINUE
```

Figure AIII.7. Stacked Splice Program (Continued)

```
      IF(XDLA/XQT-3.) 42,42,49
   42 IF(3.-XDLA/XQT) 51,53,53
 2400 CONTINUE
      IF(XDLA-3.*XQT) 57,57,51
   57 IF(3.*XQT+XDLA) 49,53,53
   53 IF(XN-XZ) 83,101,83
  101 CONTINUE
      XQTA=XQT
   83 CONTINUE
      IF(XN-XZ) 71,71,81
   88 CONTINUE
      XZA=XTDA
      XDLA=XDL
      XR=XRP*XKA(I)
      I=1
      XZB=XZA+XDLA*(XZA-XZB)/(XDLB-XDLA)
      XPA=XKA(I)*(XZB-XS(I))
       IF(XZB-XZA) 95,999,95
   71 I=1
      XPA=XR+XAM
  124 XZB=XNR(I)*XPA/XKA(I)+XS(I)
   95 XTD=XZB
      XR=XPA
      XDS=0
      XTT=-1.
      XZP=0
      XDLB=0
      XZ=0
      XQS=0
      GO TO 64
   65 CONTINUE
      I=I+1
   64 XTD=XTD-XDS
      XAS=XTD-XS(I)
      XPA=XAS*XKA(I)
      XZ=XZ+1.
      XDLB=XDLB+XPA*XNR(I)
      XSD=XDLB/XDK(I)
      XQS=XL(I)*XQL(I)+XQS
      XQT=XQS+XQI
      XQB=XQT-XDLB
      XBS=XQB/XKS(I)
      XDS=XBS-XSD
      IF(XN-XZ) 87,103,87
  103 CONTINUE
      XQTB=XQT
   87 CONTINUE
      IF(XN-XZ) 104,104,65
  104 CONTINUE
      XZ=0
      XDL=0
```

Figure AIII.7. Stacked Splice Program (Continued)

```
      XS(NS)=-XS(1)
      XKA(NS)=XKA(1)
      XNR(NS)=XNR(1)
      XAP(NS)=-XAP(1)
      NK=N*2
      XKD(NS,1)=XKD(1,1)
      XKD(NS,2)=XKD(1,2)
      I=1
      NXS=N+1
      XKA(NXS)=XKA(N)
      XS(NXS)=-XS(N)
      XNR(NXS)=XNR(N)
      XAP(NXS)=-XAP(N)
  240 CONTINUE
      XTD=0
      XDLA=0
      XDS=0
      XR=XPA
      IF(XKA(I)/XKD(I,2)-100.) 337,4338,4338
 4338 XJA(I)=XKD(I,2)
      GO TO 336
  337 XJA(I)=XKA(I)
  336 CONTINUE
      XZA=XNR(I)*XPA/XJA(I)+XS(I)
      XTLA=XR
      XTD=XZA
      GO TO 202
  201 CONTINUE
      XTD=XTD-XDS
      I=I+1
      XQT=XAP(I)*XNR(I)+XQT
  202 CONTINUE
      XAS=XTD-XS(I)
      XZ=XZ+1.
      IF(XKA(I)/XKD(I,2)-100.) 339,335,335
  335 XJA(I)=XKD(I,2)
      IF(XNN-XZ) 339,339,338
  339 XJA(I)=XKA(I)
  338 CONTINUE
      XPA=XAS*XJA(I)
      XDLA=XDLA+XPA*XNR(I)
      XSD=XDLA/XKD(I,2)
      XQB=XQT-XDLA
      XSB(I)=XQB/XKD(I,1)
      XDS=XSB(I)-XSD
  208 CONTINUE
      IF(XXT) 3333,999,340
 3333 CONTINUE
      IF(XDLA/XXT-3.) 142,142,49
  142 IF(3.+XDLA/XXT) 51,239,239
  340 CONTINUE
```

Figure AIII.7. Stacked Splice Program (Continued)

```
      XCS=C
      XDS=C
      XY=0
      I=1
  131 XTD=XZB+(XDLB-XQTB)*(XZB-XZA)/(XDLA-XQTA-XDLB+XQTB)
      XTDA=XTD
  132 XRP=XTDA
      XR=XPA
      GO TO 86
   74 CONTINUE
      I=I+1
      XTD=XTD-XDS
   86 XAS=XTD-XS(I)
      XZ=XZ+1.
      XAP(I)=XAS*XKA(I)
      XDL=XDL+XAP(I)*XNR(I)
      SXD(I)=XDL/XDK(I)
      XCS=XL(I)*XCC(I)+XCS
      XQT=XCS+XQI
      XQB=XQT-XDL
      XBS=XQB/XKS(I)
      XCS=XBS-SXD(I)
  117 IF(XN-XZ) 102,102,74
  102 CONTINUE
      IF((DABS(XDL-XQT))-.01*DABS(XAP(I))) 70,70,88
   70 CONTINUE
      XTS=0
      I=1
      XQT=C
      XST=XAP(1)*XNR(I)
      XZ=0
  204 CONTINUE
      XTV=-1.
      NT=I
      XQT=XST
      IF(XRZ) 240,318,240
  318 CONTINUE
      XPA=XKD(I,1)/(XKD(I,1)+XKD(I,2))*XST
      NAB=N-1
      DO 1500 I=1,NAB
      NK=2*N-I
      XKD(NK,2)=XKD(I,2)
      XKD(NK,1)=XKD(I,1)
      NS=2*N+1-I
      XKA(NS)=XKA(I)
      XS(NS)=-XS(I)
      XNR(NS)=XNR(I)
      XAP(NS)=-XAP(I)
 1500 CONTINUE
      I=N
      NS=2*N
```

Figure AIII.7. Stacked Splice Program (Continued)

```
      IF(XDLA-3.*XXT) 238,238,51
  238 CONTINUE
      IF(3.*XXT+XDLA) 49,239,239
  239 CONTINUE
      IF(2.*XN-XZ) 332,333,332
  333 CONTINUE
      XQTA=XQT
  332 CONTINUE
      IF(    XN-XZ) 203,203,201
  203 CONTINUE
      I=1
      XTS=0
      XPA=XR+XAM
      XZ=0
      XST=XAP(I)*XNR(I)
  206 XZB=XNR(I)*XPA/XJA(I)+XS(I)
      XTD=XZB
      XR=XPA
      XDLB=0
      XZ=0
      XQS=0
      XDS=0
      XQT=XST
      GO TO 210
  211 CONTINUE
      I=I+1
      XTD=XTD-XDS
      XQT=XAP(I)*XNR(I)+XQT
  210 XAS=XTD-XS(I)
      XPA=XAS*XJA(I)
      XZ=XZ+1.
      XDLB=XDLB+XPA*XNR(I)
      XSD=XDLB/XKD(I,2)
      XQB=XQT-XDLB
      XSB(I)=XQB/XKD(I,1)
      XDS=XSB(I)-XSD
      IF(    XN-XZ) 212,212,211
  212 CONTINUE
      XZ=0
      XDL=0
      XQS=0
      XDS=0
      XY=0
      I=1
      XTS=0
      XST=XAP(I)*XNR(I)
      XQT=XST
      XTD=XZB+XDLB*(XZB-XZA)/(XDLA-XDLB)
      XTDA=XTD
      XRP=XTDA
      GO TO 221
```

Figure AIII.7. Stacked Splice Program (Concluded)

```
  220 CONTINUE
      I=I+1
      XTD=XTD-XDS
      XQT=XAP(I)*XNR(I)+XCT
  221 XAS=XTD-XS(I)
      XZ=XZ+1.
      PXA(I)=XAS*XJA(I)
      XDL=XDL+PXA(I)*XNR(I)
      SDX(I)=XDL/XKD(I,2)
      XQB=XCT-XDL
      XSB(I)=XQB/XKD(I,1)
      XDS=XSB(I)-SDX(I)
      TP(I)=SXC(I)/XSB(I)
      XDK(I)=TP(I)*XDK(I)
      IF(2.*XN-XZ) 222,222,220
  222 CONTINUE
      DO 1001 I=1,N
      PXX=PXA(NT)
      IF(XAA-PXX) 1002,1001,1002
 1001 CONTINUE
      GO TO 250
 1002 XPA=TP(NT)*XAP(NT)
      I=1
      XAA=PXA(NT)
      XZ=0
      XZP=0
      XTT=-1.
      XIV= 1.
      GO TO 50
  250 I=1
      XZ=0
      WRITE(6,17)
      WRITE(6,19)
      XD1=0
      XD2=0
      GO TO 252
  251 CONTINUE
      I=I+1
  252 CONTINUE
      XZ=XZ+1.
      XQT=XQT+XL(I)*XQO(I)
      XD1=XD1+XAP(I)*XNR(I)-PXA(I)*XNR(I)
      XD2=XD2+PXA(I)*XNR(I)
      XBS=XQT-XD1-XD2
      WRITE(6,16) XZ,XQT,XAP(I),XD1,PXA(I),XD2,XBS
      IF(XN-XZ) 999,999,251
  999 CONTINUE
      IF(NKP-NNP) 950,951,951
  951 CONTINUE
      STOP
      END
```

Figure AIII.8. Stacked Splice Program Input Data

```
1.
  1            1
 19.
  2470000.   2470000.
  10.
-32000.
  1.0     1470000.  470000.     1000.  470000.  .0 1.  0.
  1.0     1470000.  470000.   271000.  470000.  .0 1.  0.
  1.0     1470000.  470000.   271000.  470000.  .0 1.  0.
  1.0     1470000.  470000.   271000.  470000.  .0 1.  0.
  1.0     1470000.  470000.   271000.  470000.  .0 1.  0.
  1.0     1470000.  470000.   271000.  470000.  .0 1.  0.
  1.0     1470000.  470000.   271000.  470000.  .0 1.  0.
  1.0     1470000.  470000.   271000.  470000.  .0 1.  0.
  1.0     1470000.  470000.     1000.  470000.  .0 1.  0.
  1.0     1470000.  470000.     1000.  470000.  .0 1.  0.
```

Figure AIII.9. Stacked Splice Program Output Data

SPLICE INPUT

XN= 10.

XQI=-32000.
XAED= 2470000.
XAES= 2470000.

XL	XKA	XKD1	XKD2	XKS	XS	XNR	XCO
1.00000	1470000.	470000.	1000.	470000.	0.	1.	0.
1.00000	1470000.	470000.	271000.	470000.	0.	1.	0.
1.00000	1470000.	470000.	271000.	470000.	0.	1.	0.
1.00000	1470000.	470000.	271000.	470000.	0.	1.	0.
1.00000	1470000.	470000.	271000.	470000.	0.	1.	0.
1.00000	1470000.	470000.	271000.	470000.	0.	1.	0.
1.00000	1470000.	470000.	271000.	470000.	0.	1.	0.
1.00000	1470000.	470000.	271000.	470000.	0.	1.	0.
1.00000	1470000.	470000.	1000.	470000.	0.	1.	0.
1.00000	1470000.	470000.	1000.	470000.	0.	1.	0.

SPLICE JOINT ANS

XZ	XQT	XAP1	XD1	XAP2	XD2	XBS
1.	-32000.	-14377.	-14360.	-17.	-17.	-17623.
2.	-32000.	-4173.	-12386.	-6147.	-6164.	-13450.
3.	-32000.	-844.	-12385.	-844.	-7008.	-12607.
4.	-32000.	-150.	-12410.	-125.	-7133.	-12456.
5.	-32000.	-7.	-12413.	-4.	-7137.	-12450.
6.	-32000.	109.	-12409.	104.	-7053.	-12558.
7.	-32000.	576.	-12600.	767.	-6266.	-13134.
8.	-32000.	2244.	-16545.	6188.	-78.	-15378.
9.	-32000.	-1406.	-17989.	39.	-39.	-13972.
10.	-32000.	-13972.	-32000.	39.	0.	-0.

Figure AIII.9. Stacked Splice Program Output Data (Concluded)

APPENDIX IV

COMPUTER ANALYSES DATA

APPENDIX IV

COMPUTER ANALYSES DATA

IV.1 PLASTIC DOUBLER AND SPLICE DATA

Data Set No. I	XKP (One Card) (F6.0 Format)	XKP = No. of problems To be Worked
Data Set No. II	AA, AB (One Card) (2 I10 Format)	AA = Configuration No. AB = Case No.
Data Set No. III	PLA (One Card)	PLA = 0 If Residual Load Not Desired and Positive If Desired
Data Set No. IV	XED, XES (One Card)	XED = Modulas of Elasticity of Doubler Material XES = Modulas of Elasticity of Skin Material
Data Set No. V	XN (One Card) (F6.0 Format)	XN = No. of Fastener Rows
Data Set No. VI	XW (One Card) (F6.4 Format)	XW = Density of Doubler Material
Data Set No. VII	XDTA, XWDA, XLUA (One Card) (3F10.4 Format)	XDTA = Thickness of Doubler in Front of Fastener Station 1 XWDA = Width of Doubler in Front of Fastener Station 1 XLUA = Length of Doubler in Front of Fastener 1
Data Set No. VIII	XL(I), XDT(I), XWD(I), (XN Cards) XLU(I), XTS(I), XWS(I), XS(I), XNR(I) (8 F10.4 Format)	XL(I) = Distance Shear Flow Acts on for Station I XDT(I) = Doubler Thickness for Station I XWD(I) = Effective Doubler Width for Station I XLU(I) = Distance Between Fastener Rows XTS(I) = Thickness of Base Skin at Station I XWS(I) = Effective Width of Base Skin at Station I

		XS(I) = Fastener Slop at Station I XWR(I) = No. of Fasteners in Row I.
Data Set No. IX	XQP (One Card) (F7.0 Format)	XQP = Axial Load Applied to Base Structure
Data Set No. X	XKA(I,1), XKA(I, 2) (XN Cards) XKA(I, 3) XKA(I, 4) XKA(I, 5) XKA(I, 6) (6 F11.0 Format)	XKA(I, 1) = First Fastener Spring Constant Corresponding to the First Fastener Cut Off Value at Station I XKA(I, 2) = Second Fastener Spring Constant Correspond-ing to the Second Fastener Cut Off Value at Station I XKA(I, 3) = Third Fastener Spring Constant Correspon-ing to the Third Fastener Cut Off Value at Station I XKA(I, 4) = Fourth Fastener Spring Constant Corres-ponding to the Fourth Fastener Cut Off Value at Station I XKA(I, 5) = Fifth Fastener Spring Constant Corres-ponding to the Fifth Fast-ener Cut Off Value at Station I XKA(I, 6) = Sixth Fastener Spring Constant Corres-ponding to the Sixth Fastener Cut Off Value at Station I
Data Set No. XI	XAL(I, 1), XAL(I, 2) (XN Cards) XAL(I, 3), XAL(I, 4) XAL(I, 5), XAL(I,6) (6 F10.0 Format)	XAL(I, 1) = First Fastener Cut Off Value at Station I XAL(I, 2) = Second Fastener Cut Off Value at Station I XAL(I, 3) = Third Fastener Cut Off Value at Station I XAL(I, 4) = Fourth Fastener Cut Off Value at Station I XAL(I, 5) = Fifth fastener Cut off Value at Station I XAL(I, 6) = Sixth Fastener Cut Off Value at Station I

Data Set No. XII	If PLA (DATA SET NO. III) is Positive, Requiring Residual Loads, Data Sets XII and XIII are Required if PLA is Zero, Repeat Data Sets No. II-XIII (XII and XIII for Residual Loads) for the Number of Problems to be Worked (Corresponding to Data Set No. I) XKA (I, 1) (XN Cards) (F 11.0 Format)	XKA(I, 1) = Fastener Spring Constant Corresponding to the Fastener Cut Off Value at Station I (For Residual Loads)
Data Set No. XIII	XAL (I, 1) (XN Cards) (F10.0 Format)	XAL(I, 1) = Fastener Cut Off Value at Station I (For Residual Loads) These Have To be larger than any of cut off loads for the fastener to insure the proper results. The exact number does not matter but it just has to be large to allow the routine to function properly.

Data Sets II - XIII (XII and XIII depend upon residual load requirements) are repeated for the number or problems to be worked (corresponding to Data Sets No. I).

The Plastic splice problem data is identical to the above data except Data Sets VI and VII are omitted.

IV.2 STACKED DOUBLER AND SPLICE DATA

Data Set No. I	AA, AB (One Card) (2I10 Format)	AA = Configuration No. AB = Case No.
Data Set No. II	XKP (One Card) (F6.0 Format)	XKP = No. of Problems to be worked.
Data Set No. III	XNN (One Card) (F6.0 Format)	XNN = Fastener station where spring constant of second doubler does not exist, but a dummy constant of 1000 #/in is used in program (see example stacked doubler problem) If the second doubler runs the length of the first doubler, this number is larger than the No. of fastener rows.

Data Set No. IV	XAED XAES (One Card) (2F11.0 Format)	XAED = Spring constant of doubler at first fastener station. XAES = Spring constant of Base Structure at First Fastener Station
Data Set No. V	XN (One Card) (F6.0 Format)	XN = No. of Fastener Stations
Data Set No. VI	XQI (One Card) (F7.0 Format)	XQI = Applied Axial Load
Data Set No. VII	XL, XKA, XKD1, XKD2, XKS, XS, XNR, XQO (XN Cards) (F10.5, 4F10.0, F10.3, 2F10.0 Format)	XL = Length Shear Flow act at fastener station I XKA = Fastener Spring Constant at Station I XKD1 = Spring Constant of bottom Doubler at station I XKD2 = Spring Constant of Top Doubler at station I If Top Doubler starts after Fastener Station I, place 1000 #/in into slot for a dummy spring constant. The same should be done if the top doubler ends before the bottom. XKS = Spring Constant of base structure at fastener station I XS = Slop at fastener Station I XNR = No. of fasteners at Station I XQO = Shear flow applied at Station I

The stacked splice data is identical to the stacked doubler data, except data set I and II are reversed.

All the programs are limited to 99 fastener rows because of the programs dimension statements.

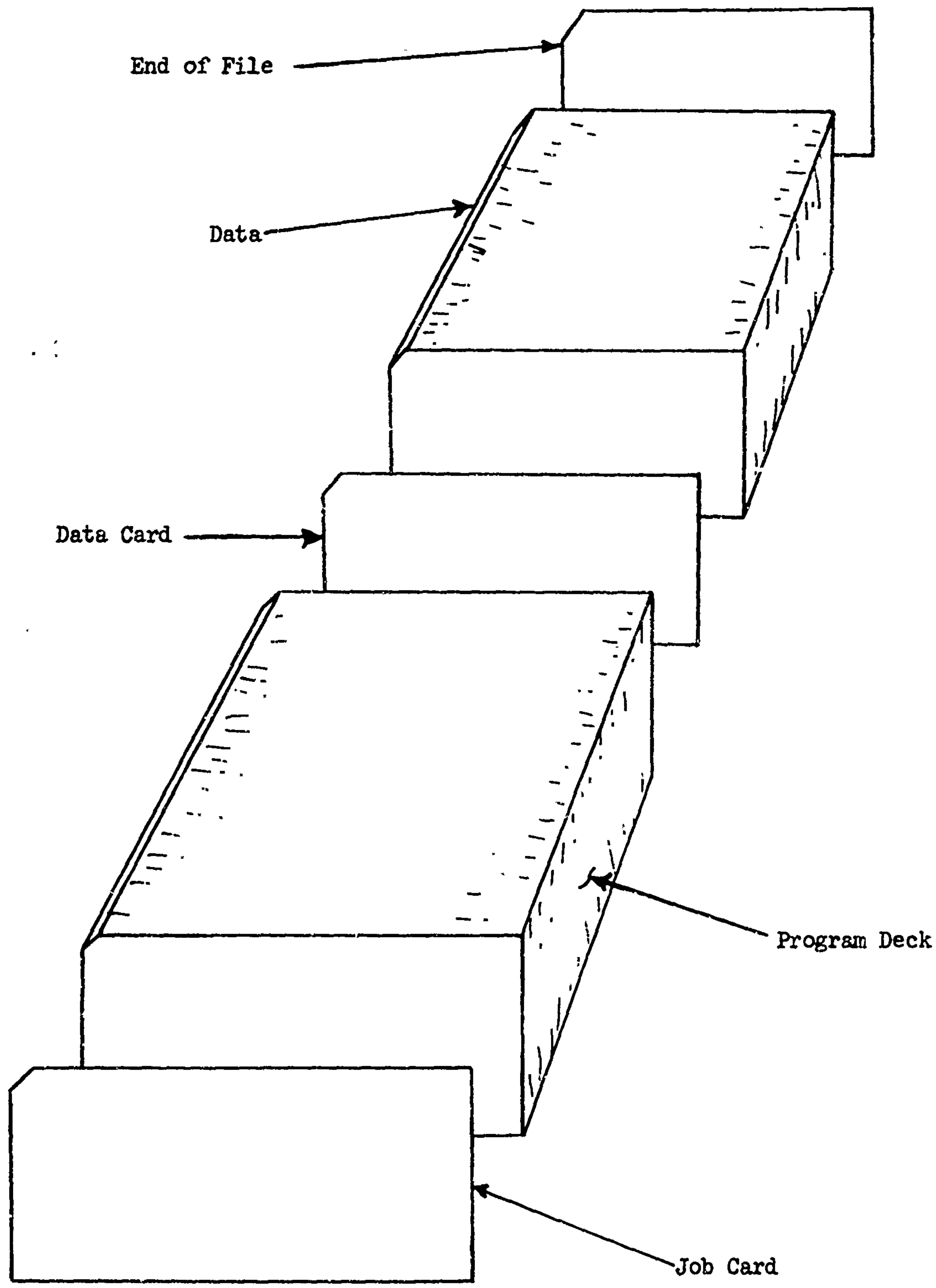


Figure AIV.1. Routine Loading Configuration

APPENDIX IV

INTERNATIONAL UNITS CONVERSION TABLE

Table AIV.1 presents the constants and instructions for converting from the English system of units into the International system of units.

TABLE AIV.1

CONVERSION FACTORS FOR THE INTERNATIONAL SYSTEM OF UNITS

To Convert From	To	Multiply By
Feet	Meters	0.3048
Feet Per Minute	Meters Per Second	0.00508
Feet Per Second	Meters Per Second	0.3048
Hours	Seconds	3600.0
Inches	Meters	0.0254
Knots	Meters Per Second	0.514444
Miles	Meters	1609.344
Pounds	Kilograms	0.4535
Minutes	Seconds	60.0
Pounds Per Square Inch (p.s.i.)	Newtons Per Square Meter	6894.7572

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13. ABSTRACT

An engineering procedure for determining the distribution of loads in the mechanically fastened joints of splice and doubler installations has been developed. Methods for both hand analyses and computer analyses are presented. Routines for solution by digital computer are provided.

The methods are generally limited to the cases of a single lap arrangement and a single sandwich arrangement, but the case of multiple (stacked) members is discussed. The members may have any form of taper or steps and the effects of fastener-hole clearance, or "slop", and plasticity can be accounted for. The particular primary data that must be supplied but which are not generally available in the literature are the spring constants of the fastener-sheet combinations.

A test program has been carried out to substantiate the methods and the results are included.

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Doubler Analyses						
Doubler Design						
Fastener Load-Deflection Data						
Fastener Load Distributions						
Splice Analyses						

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